

$$\equiv 2 \pmod{5}$$

∴ div by 2, 5, or 10: **Take the last digit**

4] Divisibility by 4 and 8

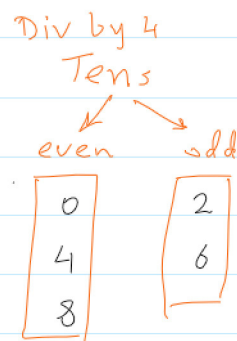
$$\text{e.g. } 2357 = 2 \times 10^3 + 3 \times 10^2 + 5 \times 10 + 7 \pmod{4}$$

Zero's

$$\equiv 57$$

$$\equiv 56 + 1 \equiv 1 \pmod{4}$$

odd b



∴ div by 4: take the last 2 digits

∴ div by 8: take the last 3 digits

5] Divisibility by 11

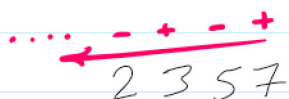
$$\text{e.g. } 2357 = 2 \times 10^3 + 3 \times 10^2 + 5 \times 10 + 7 \pmod{11}$$

$$\equiv 2(-1)^3 + 3(-1)^2 + 5(-1) + 7$$

$$\equiv -2 + 3 - 5 + 7$$

$$\equiv 3 \pmod{11}$$

∴ div by 11, **alternating +/- from right to left**



6] e.g.

$$\textcircled{1} \quad 12345 \times 5432 \pmod{11}$$

$$\text{sol}^n \equiv 3 \cdot (-2) \equiv -6 \equiv 5 \pmod{11}$$

$$\textcircled{2} \quad 12 \overset{+}{\cancel{3}} \overset{-}{\cancel{5}} \overset{+}{\cancel{5}} \overset{-}{\cancel{3}} \overset{+}{7} \equiv 6$$

$$\textcircled{3} \quad 1 \overset{-}{2} \overset{+}{\cancel{3}} \overset{-}{\cancel{5}} \overset{+}{\cancel{6}} \overset{-}{\cancel{3}} \overset{+}{7} \equiv 2$$

$$\textcircled{4} \quad \overset{-}{1} \overset{-}{\cancel{2}} \overset{-}{\cancel{3}} \overset{-}{\cancel{4}} \overset{+}{\cancel{2}} \overset{+}{\cancel{3}} \overset{+}{\cancel{4}} \overset{+}{6} \equiv 5$$

$$\textcircled{5} \quad \overset{-}{1} \overset{-}{\cancel{2}} \overset{-}{\cancel{3}} \overset{-}{\cancel{3}} \overset{-}{\cancel{2}} \overset{+}{\cancel{5}} \overset{+}{\cancel{6}} \overset{+}{1}$$

7] Divisibility by 7

e.g. $\overset{+}{6}, \overset{-}{8}, \overset{+}{1}, \overset{-}{2}, \overset{+}{3}, \overset{-}{5}, \overset{-}{6}$

$$\equiv 6 \times 10^6 + 8 \times 10^5 + 1 \times 10^4 + 2 \times 10^3 + 3 \times 10^2 + 5 \times 10 + 6 \pmod{7}$$

1	-2	-3	-1	2	-3	1
6	8	1	2	3	5	6
<u>-1</u>	<u>-2</u>	<u>-3</u>	<u>-2</u>	<u>-1</u>	<u>1</u>	<u>-1</u>
		0			-1	

$$\equiv -2$$

$$\equiv 5 \pmod{7}$$

8] e.g.

$$222,222 \equiv 0$$

$$8, \cancel{357}, \cancel{146} \equiv 0$$

$$\frac{242, 471}{-4 \quad 2} \equiv -2 \equiv 5$$

$$1 \frac{246}{246} \equiv 1$$