Matrix Representations

Tuesday, November 1, 2022 8:13 PM

Recall : Quantum Systems

§ 2.7

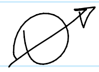
1] Classical bit : (cbit)

$$0 \text{ --- false} \longrightarrow |0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$1 \text{ --- true} \longrightarrow |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

2] Quantum bits (Qubits)

$$|\psi\rangle = \alpha_0 |0\rangle + \alpha_1 |1\rangle = \begin{bmatrix} \alpha_0 \\ \alpha_1 \end{bmatrix}$$



e.g.

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$|\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix} \longrightarrow P = \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix}$$

3] Matrix Multiplication

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} \alpha_0 \\ \alpha_1 \end{bmatrix} = \begin{bmatrix} a\alpha_0 + b\alpha_1 \\ c\alpha_0 + d\alpha_1 \end{bmatrix}$$

4] Tensor Product :

$$\begin{bmatrix} \alpha_0 \\ \alpha_1 \end{bmatrix} \otimes \begin{bmatrix} \beta_0 \\ \beta_1 \end{bmatrix} = \begin{bmatrix} \alpha_0 \begin{bmatrix} \beta_0 \\ \beta_1 \end{bmatrix} \\ \alpha_1 \begin{bmatrix} \beta_0 \\ \beta_1 \end{bmatrix} \end{bmatrix} = \begin{array}{cc} \begin{bmatrix} \alpha_0\beta_0 \\ \alpha_0\beta_1 \\ \alpha_1\beta_0 \\ \alpha_1\beta_1 \end{bmatrix} & \begin{array}{c} 00 \\ 01 \\ 10 \\ 11 \end{array} \end{array}$$

Programming

$$|\alpha\rangle \text{ --- } \quad \begin{bmatrix} P(00) \\ P(01) \\ P(10) \\ P(11) \end{bmatrix}$$

$$|\beta\rangle \text{ --- }$$

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e.g.

$$\begin{bmatrix} 1 \\ 2 \end{bmatrix} \otimes \begin{bmatrix} 3 \\ 4 \end{bmatrix} \otimes \begin{bmatrix} 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 1\cdot 3\cdot 5 \\ 1\cdot 3\cdot 6 \\ 1\cdot 4\cdot 5 \\ 1\cdot 4\cdot 6 \\ \vdots \\ 2\cdot 3\cdot 6 \\ \vdots \end{bmatrix} = \begin{array}{cc} \begin{bmatrix} 15 \\ 18 \\ 20 \\ 24 \\ \vdots \\ \vdots \\ \vdots \end{bmatrix} & \begin{array}{c} \\ \\ \\ \\ \\ 101 \\ \\ \end{array} \end{array}$$

Find : $|101\rangle = 2.3.6$

$= 36$

5] e.g.

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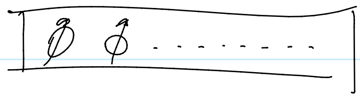
$$\begin{bmatrix}1\\0\end{bmatrix}\otimes\begin{bmatrix}0\\1\end{bmatrix}\otimes\begin{bmatrix}1\\0\end{bmatrix} = \begin{array}{c|c} \begin{bmatrix}0\\0\\1\\0\\0\\0\\0\\0\end{bmatrix} & \begin{matrix}0\\1\\2\\3\\4\\5\\6\\7\end{matrix}\end{array}$$

$= |010\rangle$

short-cut

6] Representing Multiple bits



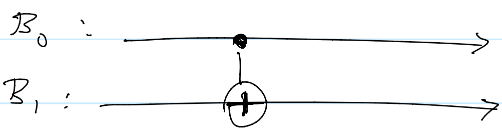
$$|00\rangle = \begin{bmatrix}1\\0\end{bmatrix}\otimes\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}1\\0\\0\\0\end{bmatrix}$$

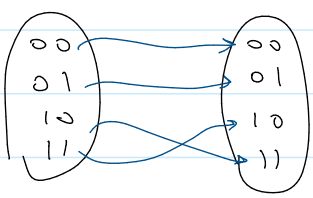
$$|01\rangle = [\ \ ]\otimes[\ \ ] = \begin{bmatrix}0\\1\\0\\0\end{bmatrix}$$

$$|11\rangle = \begin{bmatrix}0\\0\\0\\1\end{bmatrix}$$

7] Operation on multiple bits

CNOT





CNOT
$$\begin{array}{c|c} & \begin{matrix}00 & 01 & 10 & 11\end{matrix} \\ \begin{matrix}00\\01\\10\\11\end{matrix} & \begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 0 & 1\\0 & 0 & 1 & 0\end{bmatrix}\end{array}$$

8] Def^n. a vector v that can be written as a tensor of two vectors is called "separable".

8] Def^n. a vector v that can be written as a tensor of two vectors is called "separable". otherwise it is called "entangled"

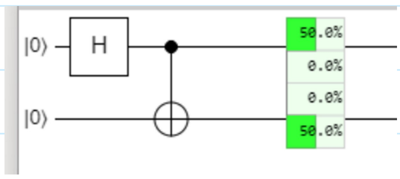
e.g. $\begin{bmatrix} 3 \\ 4 \\ 6 \\ 8 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \otimes \begin{bmatrix} 3 \\ 4 \end{bmatrix}$ is separable.

check $ad \stackrel{?}{=} bc$ $\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$

$\begin{bmatrix} 2 \\ 0 \\ 0 \\ 1 \end{bmatrix}$ is entangled

9] Bell states:

e.g. $\begin{bmatrix} 1/\sqrt{2} \\ 0 \\ 0 \\ 1/\sqrt{2} \end{bmatrix}$ is entangled



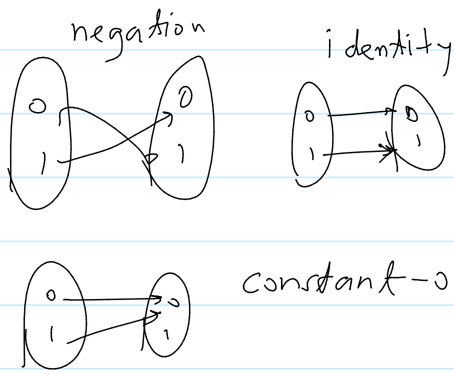
$\begin{bmatrix} 0 \\ 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{bmatrix}$ is entangled.

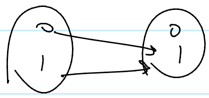
HW

10] Functions on a single bit (§ 6.1)

We have 4 functions

1. Identity : $f(x) = x$
2. negation : $f(x) = \overline{x}$
3. Constant-0 : $f(x) = 0$
4. Constant-1 : $f(x) = 1$





constant-1