

HW3

- 7.1. This exercise is about the *money change* problem stated in Exercise 6.29. Consider a currency system that has the following coins and their values: dollar (100 cents), quarter (25 cents), dime (10 cents), nickel (5 cents), and 1-cent coins. (A unit-value coin is always required). Suppose we want to give a change of value y cents in such a way that the total number of coins n is minimized. Give a greedy algorithm to solve this problem.

Algorithm: MChangeGreedy (y)

input: y

output: $C_{100}, C_{25}, C_{10}, C_5, C_1$

$$1. \quad x = y$$

$$2. \quad C_{100} = \lfloor x / 100 \rfloor$$

$$3. \quad x = x - 100 * C_{100}$$

$$4. \quad C_{25} = \lfloor x / 25 \rfloor$$

$$5. \quad x = x - 25 * C_{25}$$

$$6. \quad C_{10} = \lfloor x / 10 \rfloor$$

$$7. \quad x = x - 10 * C_{10}$$

$$8. \quad C_5 = \lfloor x / 5 \rfloor$$

$$9. \quad x = x - 5 * C_5$$

$$10. \quad C_1 = x$$

Return $C_1, C_5, C_{10}, C_{25}, C_{100}$

Algorithm MChange2

input $y, V[1 \dots n]$ // $V = [1, 5, 10, 25, 100]$

output $C[1 \dots n]$

1. $x = y$
2. for $i = n$ down to 1
3. $C[i] = \lfloor x / V[i] \rfloor$
4. $x = x - C[i] * V[i]$
5. end for
6. Return $C[1..n]$

7.2. Give a counterexample to show that the greedy algorithm obtained in Exercise 7.1 does not always work if we instead use coins of values 1 cent, 5 cents, 7 cents, and 11 cents. Note that in this case dynamic programming can be used to find the minimum number of coins. (See Exercises 6.29 and 6.30).

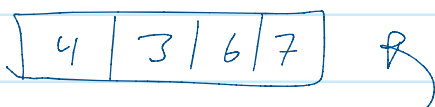
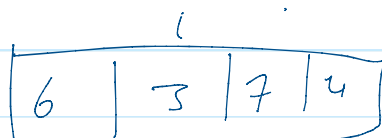
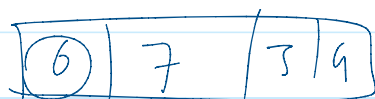
for $y = 14$

Greedy Alg: $1 \times (11) + 3 \times (1) = 14$, with 4 coins

while : $2 \times (7) = 14$, with 2 coins

\therefore the greedy algorithm is not optimal.

Split \downarrow



Algorithm 5.5 SPLIT

Input: An array of elements $A[low..high]$.

Output: (1) A with its elements rearranged, if necessary, as described above.
 (2) w , the new position of the splitting element $A[low]$.

1. $i \leftarrow low$
2. $x \leftarrow A[low]$
3. for $j \leftarrow low + 1$ to $high$
4. if $A[j] \leq x$ then
5. $i \leftarrow i + 1$
6. if $i \neq j$ then interchange $A[i]$ and $A[j]$
7. end if
8. end for
9. interchange $A[low]$ and $A[i]$
10. $w \leftarrow i$
11. return A and w

