Dynamic Programming

Saturday, October 1, 2022 1:32 PM

Dynamic Programming

Missing
2, 4, 12,

1] Fibonacci Sequence: 1, 1, 2, 3, 5, ...

2] Recursive fib : $f_n = \begin{cases} 1 & \text{for } n = 0,1 \\ f_{n-1} + f_{n-2} & \text{for } n \geq 2 \end{cases}$

Time complexity :

$$T(n) = \begin{cases} 1 & \text{if } n = 0,1 \\ T(n-1) + T(n-2) & \text{if } n \geq 2 \end{cases}$$

$$= f(n) = \alpha^n \quad , \quad \alpha = \frac{1+\sqrt{5}}{2} > 1$$

3] Dynamic Programming (fib)

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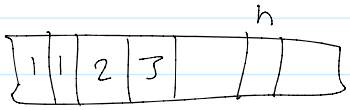
Fib(n)
A[0] = 1 , A[1] = 1
for i = 2 to n
    A[i] = A[i-1] + A[i-2]

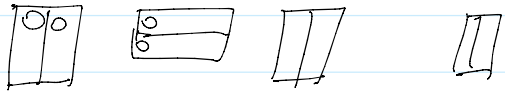
Return A[n]
```

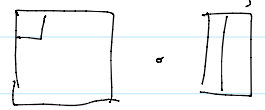
A 

4] Matrix-Chain Multiplication

$M_1 \cdot M_2 \cdot M_3 = M$

e.g. $(10 \times 2) \cdot (2 \times 10) \cdot (10 \times 2) \quad (10 \times 2)$



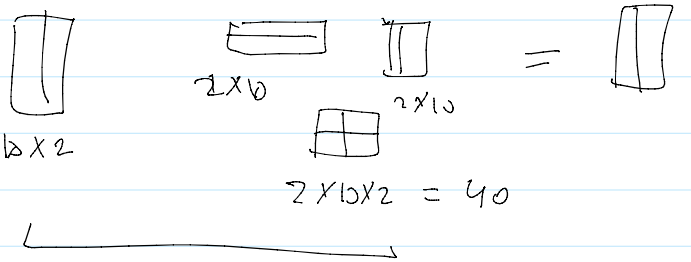
① $M_1 M_2$ (10×10)  $\cdot$ $=$  $\Rightarrow$ 400 mult ops.

$10 \times 2 \times 10 = 200$

$(10 \times 10) \quad (10 \times 2)$

$10 \times 10 \times 2 = 200$

② $M_1 \cdot (M_2 \cdot M_3)$



10×2 $\quad 2 \times 10$ $\quad 2 \times 10$

$2 \times 10 \times 2 = 40$

$10 \times 2 \times 2 = 40$

$\Rightarrow$ 80 mult ops.

5] Matrix - Chain Problem:

What is the minimum cost of multiplying a chain of n matrices

6] How many ways to parenthesize n matrices?

Let $f(n)$ be number of ways to multiply n matrices.

$M = \underbrace{(M_1 \cdot M_2 \cdot M_3 \cdot \cdots \cdot M_k)}_{f(k)} \cdot \underbrace{(M_{k+1} \cdot \cdots \cdot M_n)}_{f(n-k)}$

$$f(n) = \sum_{k=1}^{n} f(k) \cdot f(n-k) = \frac{1}{n} \binom{2n-2}{n-1} = C_{n-1}$$

$$f(n) = \Omega\left(4^n / n^{1.5}\right)$$

$$\binom{n}{r} = \frac{n!}{(n-r)!\,r!}$$

Quiz 1.

$$T(n) = 3\,T\left(\frac{n}{9}\right) + \sqrt{5n}$$

$$\log_9 3 = 1/2$$

$$f(n) = \sqrt{5n} = \Theta(n^{1/2}) \implies \text{case } ②$$

$$\therefore \text{by MT. case 2, } T(n) = \Theta(n^{1/2} \cdot \log n)$$

$$\log_b a = \frac{\log a}{\log b}$$

$$\log_2 n = \log_{10} n \times 3.32$$

#2.

i	A[i]	3	9	5	2	4	7	8	C	A
			↓							
2	9	3	9						1	2
3	5	3	5	9					2	3
		2	3	5	9				3	5
		2	3	4	5	9			3	4
		2	3	4	5	7	9		2	3
							8	9	2	3
									13	20