Review

Friday, September 30, 2022 4:17 PM

Quiz B1

How many comparisons does the insertion sort need to sort this array?

A = [3, 5, 1, 8, 2, 6]

i	a
2	5
3	1
4	8
5	2
6	6

3 5 1 8 2 6

3 5

1 3 5

1 3 5 8

1 2 3 5 8

1 2 3 5 6 8

C	A
1	2
2	4
1	2
4	5
2	3
10	16

How many comparisons does the Button-up Mergesort need to sort this array

A = [2, 4, 3, 5, 6, 7]

2 4 3 5 6 7

2 4 3 5 6 7

2 3 4 5

2 3 4 5 6 7

C	A
3	12
3	8
4	12
10	32

Let T(n) = 5T(n/6) + n / log n.

Apply the master theorem to give an asymptotic bound for T(n) using an appropriate notation (O, Omega, or Theta).

$$R = \log_6 5 \cong 0.9$$

$$\frac{n}{\log n} = \Omega(n^{R+\varepsilon})$$

$\Rightarrow$ Case :

$$T(n) = \Theta(n/\log n)$$

HW 2

Exer 24.

$n! = \Theta(n^n)$ False $n! = o(n^n)$

$n! = 1 \cdot 2 \cdot 3 \cdot \underbrace{\cdots \cdot n}$

$< 1 \cdot 2 \cdot 3 \cdot n \cdot n \cdots n$

$= 6 \cdot n^{n-3} = o(n^3 \cdot n^{n-3}) = o(n^n)$

Note: $\log(n!) = \Theta(\log n^n) = \Theta(n \log n)$

1.15. Express the following functions in terms of the Θ-notation.

(a) $2n + 3\log^{100} n.$ $= \Theta(n)$

(b) $7n^3 + 1000n\log n + 3n.$ $= \Theta(n^3)$

(c) $3n^{1.5} + (\sqrt{n})^3 \log n.$ $= \Theta(n^{1.5} \log n)$

(d) $2^n + 100^n + n!.$ $= \Theta(n!)$

1.16. Express the following functions in terms of the Θ-notation.

(a) $18n^3 + \log n^8.$ $= \Theta(n^3)$

(b) $(n^3 + n)/(n + 5).$ $= n^2 + \cdots = \Theta(n^2)$

(c) $\log^2 n + \sqrt{n} + \log\log n.$ $= \Theta(\sqrt{n})$ $(\log n)^2$

(d) $n!/2^n + n^n.$ $= \Theta(n^n)$

(e) $n! + n^n/2^n$ HW $= \Theta\left(\left(\frac{n}{2}\right)^n\right)$

	$n!$	$n^n/2^n$
$n = 5$		
$n = 6$	720	$\frac{6^6}{64} = 729$

1.22. Show that $n^{100} = O(2^n)$, but $2^n \neq O(n^{100})$.

1.23. Show that 2^n is not $\Theta(3^n)$.

$2^n = O(3^n)$

$n^{100} < 2^n$

$n \quad 7$

$\lim_{n \to \infty} \frac{(2^7)^{100}}{2^{7 \cdot 100}} = \infty$

for $n = 2 \cdot 100$

$n = (2^{14})$

$$2^{100} \cdot 100^{100} < 2^{100} \cdot 2^{100}$$

$$n = (2^7)^{100}$$

$$(2^{700})^{100} < 2^{(2^{700})}$$

$$= 2^{70000}$$

$$\log n \quad \text{vs} \quad n^{0.01}$$

$$n = 10^{100} \qquad (10^{100})^{0.01} = 10$$

$$100$$

1.29. Use the $\prec$ relation to order the following functions by growth rate: $\underbrace{n^{1/100}}_{6}$, $\underbrace{\sqrt{n}}_{7}$, $\underbrace{\log n^{100}}_{4}$, $\underbrace{n \log n}_{8}$, $\underbrace{5}_{2}$, $\underbrace{\log \log n}_{3}$, $\underbrace{\log^2 n}_{5}$, $\underbrace{(\sqrt{n})^n}_{9}$, $\underbrace{(1/2)^n}_{1}$, $\underbrace{2^{n^2}}_{11}$, $\underbrace{n!}_{10}$.

$$(\sqrt{n})^n = (n^{1/2})^n = n^{n/2} = n \cdot n \cdot n \cdots n$$

$$n! = n \cdot (n-1) \cdots (n/2) \cdots (1)$$

$$2^{(n^2)} \quad n^n$$

. ange(1, n+1)

$\log^2 n$

6

$\log n$

$6 \log^3 n$

$$6 \times \frac{n(n+1)(2n+1)}{6}$$

2
4
8
16
32
64
128
256
512

$2^{10} = 1024$