Cayley Theorem

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Recall: Permutation group.

13] Thrm (Cayley Theorem)

[13] Theorem. (Cayley Theorem) Every group is isomorphic to a subgroup of some permutation group.

$2 \longrightarrow 4 \longrightarrow 3 \longrightarrow 1$

14]

[14] Examples.

(a) The additive group $\mathbf{Z}_2$ is isomorphic to the permutation group S_2, with the trivial isomorphism mapping of $\theta(0) = [1\ 2]$ and $\theta(1) = [2\ 1]$.

(b) The additive group $\mathbf{Z}_3$ is isomorphic to a subgroup of S_3, with the isomorphism mapping of $\theta(0) = [1\ 2\ 3]$, $\theta(1) = [2\ 3\ 1]$, and $\theta(2) = [3\ 1\ 2]$.

(c) The multiplicative group $\mathbf{Z}^*_5$ is isomorphic to a subgroup of S_4. Here, we notice that ord(2) = 4. So we need a permutation of order 4 to generate a subgroup of 4 elements. Taking the permutation [2 3 4 1], we obtain a possible mapping as follows: $\theta(1) = [1\ 2\ 3\ 4]$, $\theta(2) = [2\ 3\ 4\ 1]$, $\theta(3) = [4\ 1\ 2\ 3]$, and $\theta(4) = [3\ 4\ 1\ 2]$.