Recall: S_n

$|S_n| = n!$

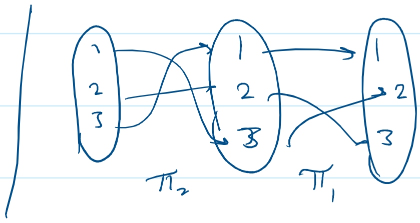
6] e.g. $(S_3, \circ)$

$A = \{1, 2, 3\}$

$S_3 = \{\ \pi_0 = [1\ 2\ 3],\ \overset{\pi_1}{[1\ 3\ 2]},\ \overset{\pi_2}{[2\ 1\ 3]},$
$[2\ 3\ 1],\ [3\ 1\ 2],\ [3\ 2\ 1]\ \}$

e.g. $[1\ 3\ 2] \circ [3\ 2\ 1] = [2\ 3\ 1] \in S_3$

$\overset{\pi_4}{[3\ 1\ 2]} \circ \overset{\pi_3}{[2\ 3\ 1]} = [1\ 2\ 3]$



$\pi_4 \circ \pi_3 = \pi_0$

$\Longrightarrow \pi_4^{-1} = \pi_3$

e.g. Find $\pi_3^{-1} = [2\ 3\ 1]^{-1}$
$= [3\ 1\ 2] = \pi_4$

$\pi_5^{-1} = [3\ 2\ 1]^{-1} = [3\ 2\ 1] = \pi_5$

7] Thrm: $\forall\, \pi \in S_n$, π can be written as a product of disjoint cycles. The cycles here are unique.

8] e.g. $A = \{1, 2, 3, 4, 5, 6\}$

let $\pi = [1\ 4\ 6\ 5\ 2\ 3] \in S_6$

<u>Arrow notation</u> — Cycle notation

$c_1 : 1 \to 1$ — (1)

$c_2 : 2 \to 4 \to 5 \to 2$ — $(2\ 4\ 5)$

$c_3 : 3 \to 6 \to 3$ — $(3\ 6)$

$$\pi = c_1 \circ c_2 \circ c_3 = (1) \circ (2\ 4\ 5) \circ (3\ 6)$$

$$\pi(2) = c_1 \circ c_2 \circ c_3(2) = c_1(c_2(c_3(2))) = c_1(c_2(2)) = c_1(4) = 4$$

$$\pi(6) = c_1 \circ c_2 \circ c_3(6) = c_1(c_2(3)) = 3$$

9] Notation: for compactness

① use juxtaposition $\Longrightarrow \pi = (1)(2\ 4\ 5)(3\ 6)$

② omit unit cycles $\Longrightarrow \pi = (2\ 4\ 5)(3\ 6)$

③ for identity: $\pi_0 = (1)(2)(3)(4)(5)(6) = (1)$

10] Prop. if $\pi \in S_n$ is given as a product of cycles, then ord(π) is the lcm(the lengths of the cycles)

11] e.g. $\pi = [1\ 4\ 6\ 5\ 2\ 3] = c_1 \circ c_2 \circ c_3 = \underline{(2\ 4\ 5)}\,\underline{(3\ 6)}$

$\pi^2 = [1\ 5\ 3\ 2\ 4\ 6] =$

$\pi^3 = [1\ 2\ 6\ 4\ 5\ 3] =$

$\pi^4 = [1\ 4\ 3\ 5\ 2\ 6] =$

$\pi^5 = [1\ 5\ 6\ 2\ 4\ 3]$

$\pi^6 = [1\ 2\ 3\ 4\ 5\ 6]$

$\text{ord}(\pi) = \text{lcm}(1, 3, 2) = 6$

12] e.g. In S_7, find a subgroup of

① order 3.

take a cycle of length (2 3 1)

let $\pi = [2\ 3\ 1\ 4\ 5\ 6\ 7]$

$\therefore H = \{\pi, \pi^2, \pi^3\}$

$= \{[\underline{2\ 3\ 1}\ 4\ 5\ 6\ 7], [3\ 1\ 2\ 4\ 5\ 6\ 7],$

$[1\ 2\ 3\ 4\ 5\ 6\ 7]\}$

② order 10

take 2 cycles $c_1 = (2\ 3\ 4\ 5\ 1)$, $c_2 = (7\ 6)$

find π of order 10

let $\pi = [2\ 3\ 4\ 5\ 1\ 7\ 6]$

$\therefore H = \{\pi, \pi^2, \pi^3, \pi^4,$

$\ldots \pi^{10} = \pi_0\}$

$H = \{ [\underline{2\ 3\ 4\ 5\ 1}\ \underline{7\ 6}],$

$[3\ 4\ 5\ 1\ 2\ \underline{6\ 7}],$

$[4\ 5\ 1\ 2\ 3\ 7\ 6],$

$[5\ 1\ 2\ 3\ 4\ \underline{6\ 7}],$

$[\underline{1\ 2\ 3\ 4\ 5}\ 7\ 6],$

$[2\ 3\ 4\ 5\ 1\ \underline{6\ 7}],$

$[3\ 4 \ldots\ 7\ 6],$

$[4\ 5 \ldots\ \underline{6\ 7}],$

$[5\ 1 \ldots\ 7\ 6],$

$[\underline{1\ 2\ 3\ 4\ 5}\ \underline{6\ 7}]\}$