# Decision Tree Model

Note Title 10/1/2019

## Recall : Decision Tree Model

1] Sorting Problem

How to sort?

by comparison

radix / buckets (excluded)

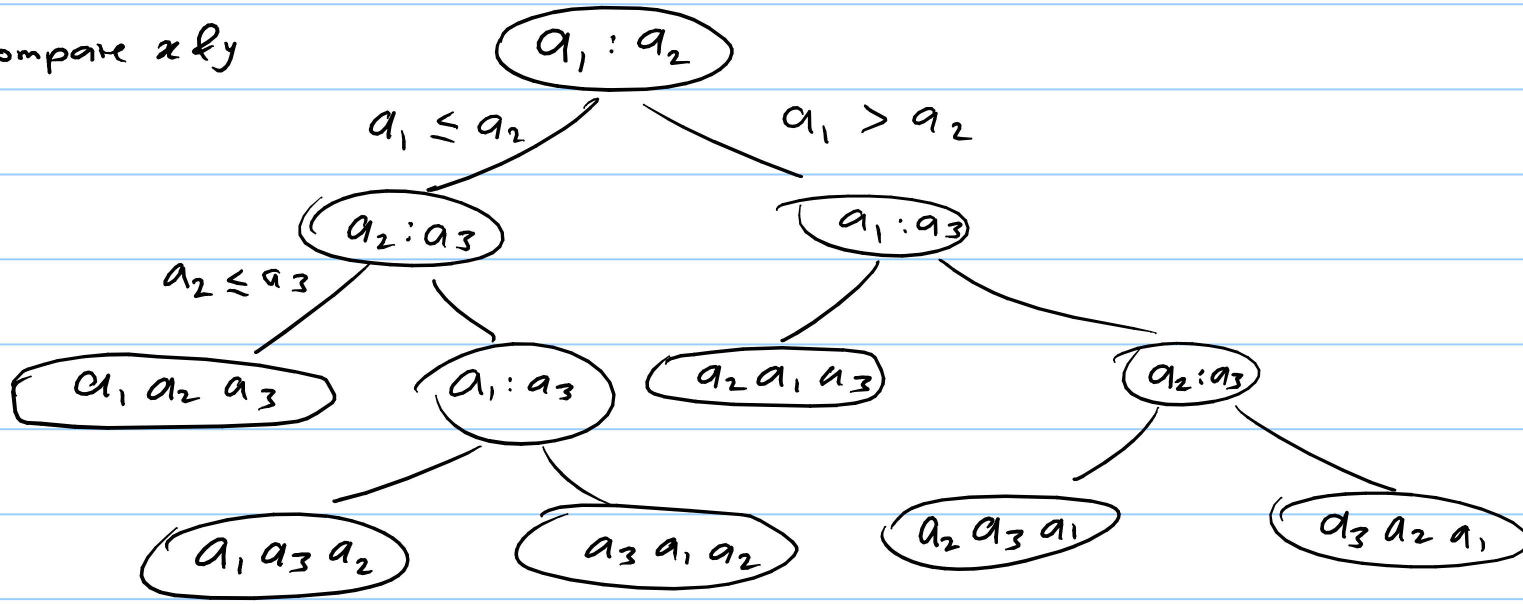
<u>DT Model</u>:

↳ 1) each internal node is a comparison

↳ 2) each leaf-node is an output (permutation of the input)

2] e.g. Decision tree for sorting : $a_1 \; a_2 \; a_3$

$x : y \equiv$ compare $x$ & $y$




3] How many leaf-node?

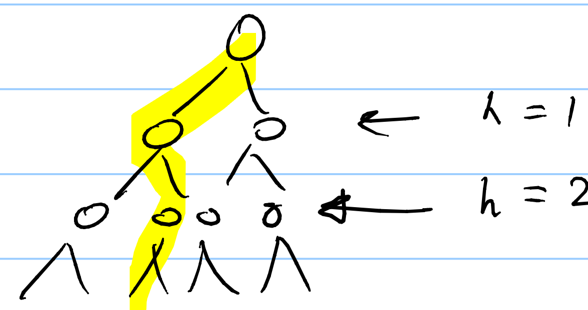
$$l \ge n!$$

4] What is the height of the decision tree of sort?

Let $l$ = number of leaves in $T$

$h$ = height of $T$

At level $h$, we have $l \leq 2^h$




$\therefore \quad n! \leq l \leq 2^h$

$\Rightarrow \quad h \geq \log(n!) = \Theta(n \log n)$

$\therefore$ Number of comparisons $= h = \Omega(n \log n)$

$\Rightarrow$ merge sort and heap-sort are optimal.

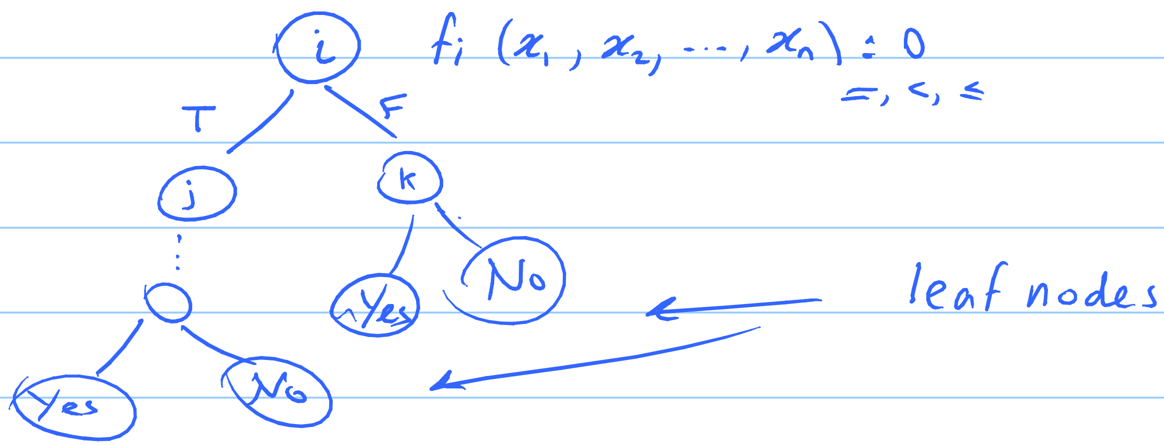
## §11.4 Algebraic Decison Tree (adt)

5] **Def$^n$.** let $\Pi(x_1, x_2, \cdots, x_n)$ is a decision problem whose input is set of $n$ variables.

The adt is a binary tree s.t.

1) each internal node $i$ is of the form $f_i(x_1, x_2, \cdots, x_n) : 0$
2) each leaf-node is either "Yes" or "No" output.

$f_i : 0$ denotes one of the following comparisons $(=, <, \leq)$



6] The order of an adt is $d$ if adt $f_i$ are polynomials of degrees $\leq d$

7] a Linear decision tree (ldt) is an adt of degree 1.

8] let $E^n$ be an n-dimentional space, $W \subseteq E^n$ s.t.
$(x_1, x_2, \dots, x_n) \in W$ <u>iff</u> $\Pi(x_1, x_2, \dots, x_n) = Yes$

9] We say an adt $T$ decides the membership in $W$ if when we start at the root of $T$ with the point $p = (x_1, x_2, \dots, x_n)$, the control will reach a "Yes" leaf node iff $p \in W$.

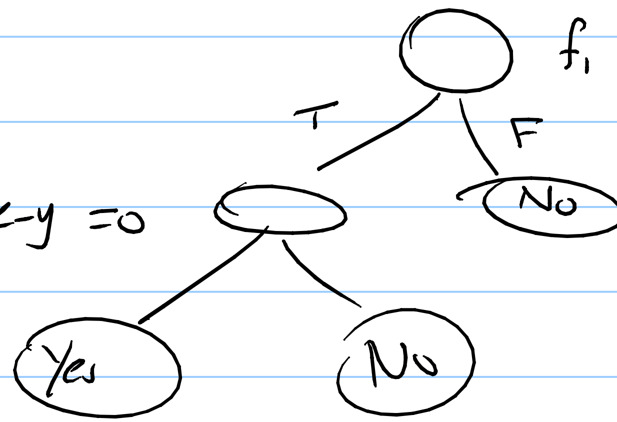
10] Notations: $\#W$ denotes the number of connected components of the set $W$.

e.g. The black cable on the floor.

T:

$f_1(x, y, z) = z = 0$

$f_2(x, y, z) = x - y = 0$

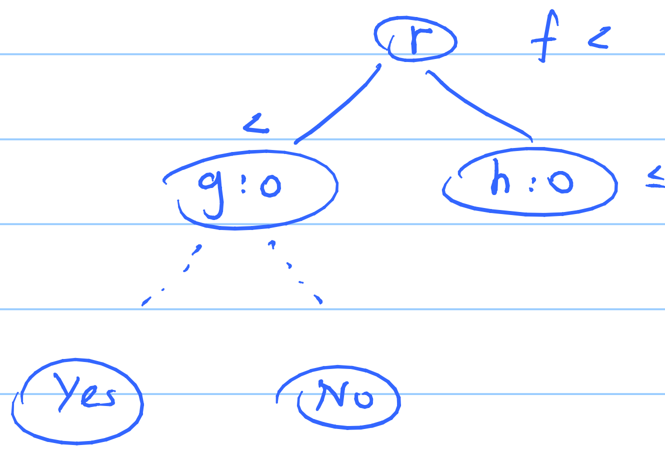


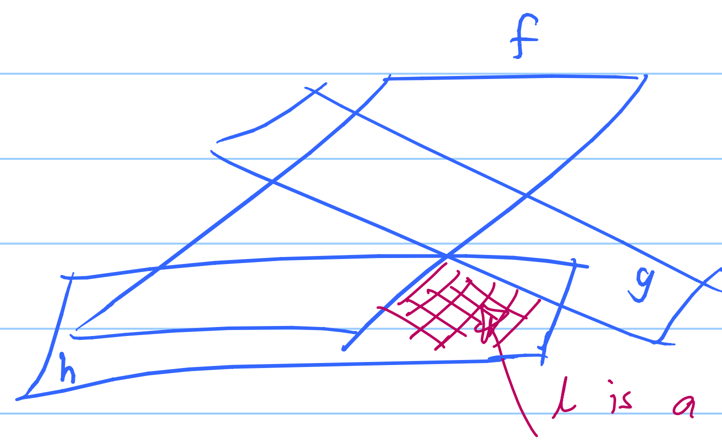
11] Deriving lower bounds using adt / ldt model

1. build an adt / ldt $T$ for $\Pi$
2. find the lower bound on the height of $T$.

e.g. ldt Model

let $T$ be an ldt for $\Pi$




$l$ is a convex-set

$l$ is connected

$\# l = 1$