Permutation Groups

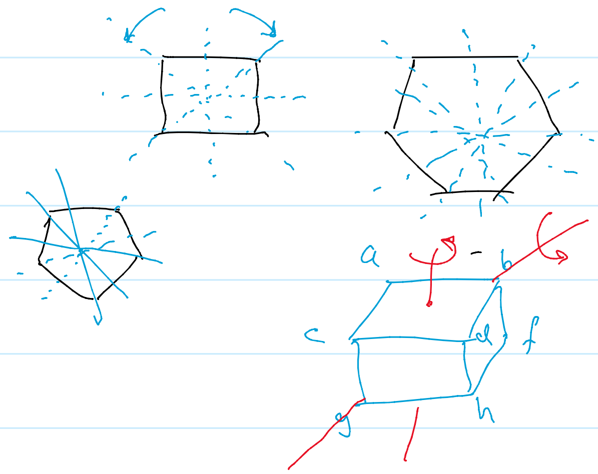
Monday, March 8, 2021 8:34 AM

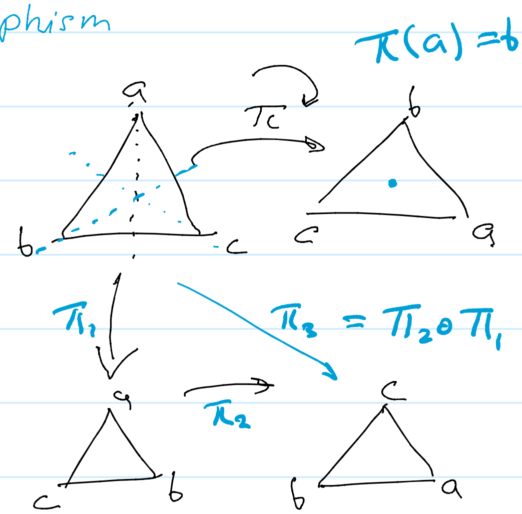
Recall : Cyclic groups

$G = \langle \alpha \rangle \Rightarrow \langle \alpha^k \rangle = G$ iff $\gcd(k, |G|) = 1$

$\Rightarrow G$ has $\Phi(|G|)$

Group Isomorphism





1] Def^n. let $A = \{1, 2, \dots, n\}$

a permutation π of A is a bijection from A to A

$\pi : A \to A$

$A : x$	1	2	3	..	n
$\pi(x)$	2	3	7	....	1

$\longrightarrow \pi = [2\ 3\ 7 \cdots 1]$

2] Notations : let $A = \{1, 2, \dots, n\}$

$S(A)$ is the set of all permutations of A

let $n = |A|$

$S_n = S(A) = \{\pi \mid \pi$ is a permutation of n elements$\}$

3] e.g. for $n = 3$, $A = \{1, 2, 3\}$

x	1	2	3
$\pi(x)$	1	3	2

$S_3 = \{[1,2,3], [1,3,2], [2,1,3],$

$[2,3,1], [3,1,2], [3,2,1]\}$

$|S_3| = 6 = 3!$

4] Prop. $(S_n, \circ)$ is a **group** with the operation of function composition $\pi_1 \circ \pi_2$ defined by

$$\forall x \in A, \quad \pi_1 \circ \pi_2 (x) = \pi_1(\pi_2(x))$$

Here, S_n is the permutation group of A

a.k.a. the symmetric group of A

5] Note: In $(S_n, \circ)$, $\pi_1 \circ \pi_2$ is read π_1 after π_2

the identity is the neutral permutation e, $e(x) = x \ \forall x \in A$.

S_n is non-abelian for $n > 2$

for $S_2 = \{[1\ 2], [2\ 1]\}$ is abelian.

6] e.g. $(S_3, \circ)$

$$S_3 = \{[1,2,3], [1,3,2], [2,1,3], [2,3,1], [3,1,2], [3,2,1]\}$$

e.g. $\underbrace{\overset{1\ \ 2\ \ 3}{[1\ 3\ 2]}}_{\pi_1} \circ \underbrace{\overset{1\ \ 2\ \ 3}{[3\ 2\ 1]}}_{\pi_2} = [2\ 3\ 1] \in S_3$

$e = [1\ 2\ 3]$

e.g. $[1\ 2\ 3] \circ [3\ 1\ 2] = [3\ 1\ 2]$

Quiz 3 → Sat 4PM.