

Greatest Common Divisor

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6:55 PM

1] $\gcd(a, b) = d$ if

① $d \mid a$ and $d \mid b$

and ② $\forall c$, if $c \mid a$ and $c \mid b$, $d \geq c$

2] $\forall a$, $\gcd(a, 0) = |a|$

Note $\gcd(0, 0) = 0$ by defⁿ.

3] e.g. $\gcd(80, -24) = 12$
 $\gcd(-6, 0) = 6$

4] Find $\gcd(424, 282)$

sol.

by factorization

$$424 = 2^4 \cdot 53$$

$$282 = 2 \cdot 3 \cdot 47$$

$$\gcd = 2^1 = 2$$

$$\begin{array}{r|l} 2 & 424 \\ \hline 2 & 212 \\ 2 & 106 \\ 2 & 53 \\ 53 & 1 \end{array}$$

$$\begin{array}{r|l} 2 & 282 \\ \hline 3 & 141 \\ 47 & 47 \\ & 1 \end{array}$$

5] LCM: Least Common Multiple

Defⁿ. Let $a, b > 0$

$\text{lcm}(a, b)$ is the smallest positive integer m
s.t. $a \mid m$ and $b \mid m$

e.g. Find $\text{lcm}(24, 60)$
n.s. $2^3 \cdot 3^1$

$$\begin{array}{r|l} 2 & 24 \\ \hline 2 & 12 \end{array}$$

$$\begin{array}{r|l} 2 & 60 \\ \hline 2 & 30 \end{array}$$

e.g. Find lcm (24, 60)

$$24 = 2^3 \cdot 3^1$$

$$60 = 2^2 \cdot 3^1 \cdot 5^1$$

$$\text{lcm} = 2^3 \cdot 3^1 \cdot 5^1 = 120$$

$$\begin{array}{r|l} 2 & 24 \\ \hline 2 & 12 \\ 2 & 6 \\ 3 & 3 \\ & 1 \end{array}$$

$$\begin{array}{r|l} 2 & 60 \\ \hline 2 & 30 \\ 3 & 15 \\ 5 & 5 \\ & 1 \end{array}$$

6] Euclidean Algorithm

Thm: $\text{gcd}(a, b) = \text{gcd}(b, a \bmod b)$

e.g.

Find $\text{gcd}(414, 248)$

$$414 = 1 \cdot 248 + 166$$

$$248 = 1 \cdot 166 + 82$$

$$166 = 2 \cdot 82 + 2$$

$$82 = 41 \cdot 2 + 0 \quad \rightarrow \text{gcd}$$

$$\begin{array}{l} \text{gcd}(414, 248) \\ (248, 166) \end{array}$$

7] Thm:

$$\text{gcd}(a, b) \cdot \text{lcm}(a, b) = a \cdot b$$

e.g. Find $\text{lcm}(414, 248)$

$$\text{lcm} = \frac{414 \cdot 248}{\text{gcd}(414, 248)} = \frac{414 \cdot 248}{2}$$

$$\begin{array}{l} \text{gcd}(a, b) \\ a = 2^{a_1} \cdot 3^{a_2} \cdot 5^{a_3} \cdot 7^{a_4} \dots \\ b = 2^{b_1} \cdot 3^{b_2} \cdot 5^{b_3} \cdot 7^{b_4} \dots \\ \text{gcd} = 2^{\min(a_1, b_1)} \cdot 3^{\min(a_2, b_2)} \dots \end{array}$$

$$\text{lcm} = 2^{\max}$$

$$\text{lcm} \cdot \text{gcd} = \frac{a_1 \cdot b_1}{2 \cdot 2}$$