NFA

Tuesday, April 19, 2022 9:40 PM

Recall: FSA

DFA ↙ ↘ NFA

§4. Nondeterministic Finite Automata (NFA)

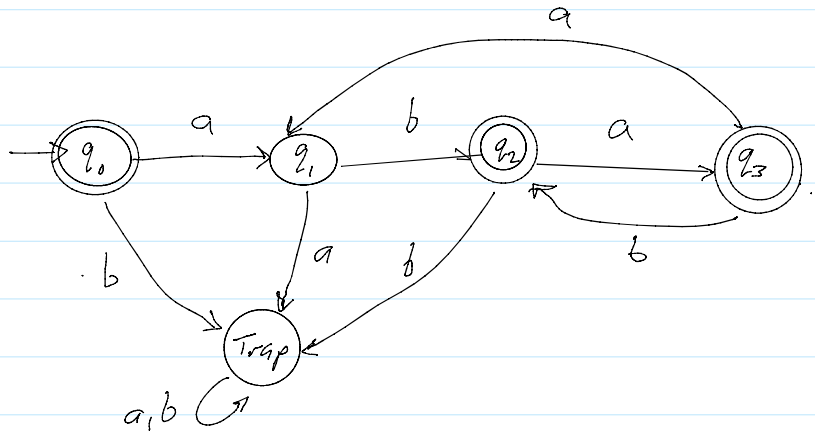
1) Two main differences between DFA and NFA

1. NFA have multiple next-states on the same input.
   (zero or more next states)
2. NFA allow λ-transitions

e.g.

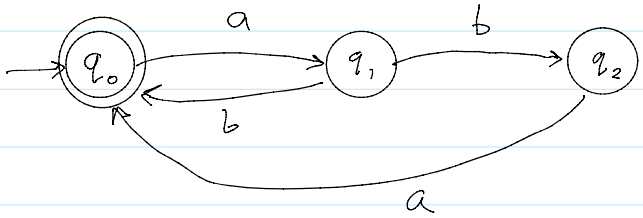
$$L = (ab + aba)^*$$

DFA:

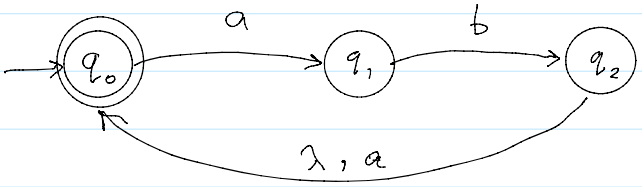


NFA ① (multiple next states) $L = (ab + aba)^*$

NFA



NFA ② (λ-transition)



2] Def^n. a nondeterministic finite automata (NFA) is a quintuple

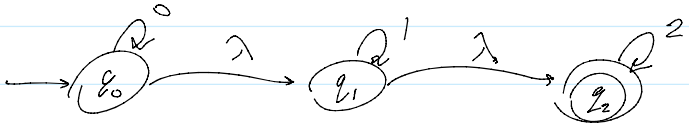
$$M = (Q, \Sigma, \delta, s, F), \text{ where}$$

Q is a finite set of states
Σ is the alphabet
$\delta : Q \times (\Sigma \cup \{\lambda\}) \to 2^Q$ is the transition function
$s \in Q$ is the start state
$F \subseteq Q$ is the set of final states.

$2^Q = \{X \mid X \subseteq Q\}$ power set.

e.g. Describe the NFA M_3 formally

M_3 :



Sol-

$$M_3 = (Q, \Sigma, \delta, q_0, \{q_3\})$$

$Q = \{q_0, q_1, q_2\}$,
$\Sigma = \{0, 1, 2\}$

$\delta : Q \times (\Sigma \cup \{\lambda\}) \to 2^Q$

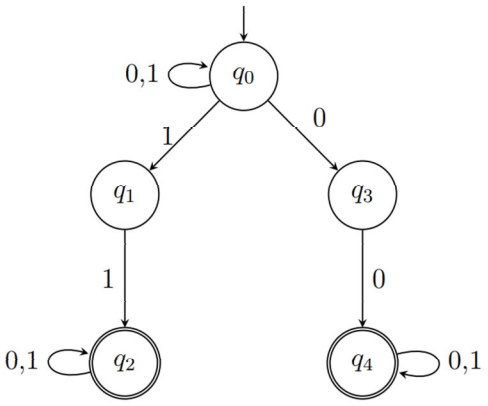
δ	0	1	2	λ
q_0	$\{q_0\}$	$\{\}$	$\{\}$	$\{q_1\}$
q_1	$\{\}$	$\{q_1\}$	$\{\}$	$\{q_2\}$
q_2	$\{\}$	$\{\}$	$\{q_2\}$	$\{\}$

e.g. $002 \in L(M_3)$?

Yes, for $q_0 \xrightarrow{0} q_0 \xrightarrow{0} q_0 \xrightarrow{\lambda} q_1 \xrightarrow{\lambda} q_2 \xrightarrow{2} q_2$

e.g.

M_4



$\left.\begin{matrix} 000 \\ 11100 \end{matrix}\right\} \in L(M_4)$

is 01001 accepted by M_4? Show state-propagation

$q_0 \xrightarrow{0} q_0 \xrightarrow{1} q_0 \xrightarrow{0} q_0 \xrightarrow{0} q_0 \xrightarrow{1} q_0$

$q_0 \xrightarrow{0} q_3$, $q_0 \xrightarrow{1} q_1$, $q_0 \xrightarrow{0} q_3$, $q_0 \xrightarrow{0} q_3$, $q_0 \xrightarrow{1} q_1$

$q_3 \xrightarrow{0} q_4 \xrightarrow{1} q_4 \in F$

$\therefore$ 01001 is accepted at $q_4 \in F$

3] Note: the extended transition function in NFA

$$\delta^* : Q \times \Sigma^* \longrightarrow 2^Q$$

$\delta^*(q, x)$ is a set of all states reachable from q after reading string x

e.g. in M_4, $\delta^*(q_0, 01001) = \{q_0, q_1, q_4\}$

4] The language of an NFA $M = (Q, \Sigma, \delta, q_0, F)$ is

$$L(M) = \{w \mid \delta^*(q_0, w) \cap F \neq \emptyset\}$$

5] Note: DFA is a special type of NFA.

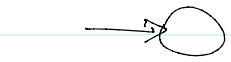
5] Note : DFA is a specialty of ...
i.e. every DFA is an NFA

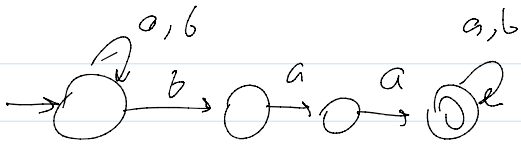
6] Thm : Ever NFA is equivalent to some DFA that accepts the same language.

7] Thm : the languages of NFA are regular.

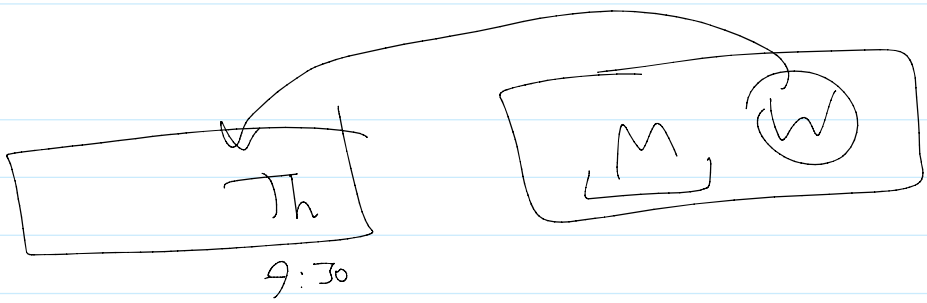
Exer:

L	RE	NFA
①	all strings	$(a+b)^*$
②	$\emptyset$	
③	contains "baa" as a substring	





END of LNTC



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