Regular Languages

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Recall : Regular Expressions

RE is a string on Δ

$\Delta = \Sigma \cup \{*, +, (,), \emptyset\}$, such that

1. $\emptyset$ and $\forall a \in \Sigma$ are the basic REs.
2. if α is RE, then so is α^* (star)
3. if α and β are REs, then so is $\alpha\beta$ (concatenation)
4. if α and β are REs, then so is $\alpha+\beta$ (union)

e.g.

$$L((ab)^*) = \{\lambda, ab, abab, ababab, \dots\}$$

$$L(ab^*) = \{a, ab, abb, abbb, \dots\}$$

2] Precedence rules: $*$, concatenation, union.

$$L(ab^* + b) = \{a, b, ab, abb, \dots\}$$

3] Notation: in Rule 4, the union operation '+' means "or", alternatively some textbooks use "|", "U"

We will use + by default.

4] Notation: α is a RE

$L(\alpha)$ denotes the language represented by α

e.g.

$\alpha = a(a+b)^*$ $\quad L(\alpha)$ = all strings start in a

$\beta = (ab^*)^*$ $\quad L(\beta)$ = all strings start in a

$$L(\alpha) = L(\beta) \iff \alpha \equiv \beta$$

We can do $\alpha = L(\alpha) = \beta$ if we talk about the languages

Exer:

Language L	$x \in L$	$y \notin L$	RE α

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Language L	$x \in L$	$y \notin L$	RE α
① start in 1, end in 0 on $\{0,1\}$	10, 10010	101	$1(0+1)^*0$
② all strings on $\{a,b\}$	aa, bbb	none	$(a+b)^* \equiv (a^*b^*)^*$
③ $\{\ \} = \emptyset$	none	ab, a, λ	$\emptyset$
④ $\{\lambda\}$	λ	ab, a	$\emptyset^*$
⑤ $\{w \mid w$ has even a's$\}$	b, aa, aba, baaaabb	abb,	$b^*(b^*ab^*ab^*)^* \equiv b^*(ab^*ab^*)^*$
⑥ $\{w \mid w$ ends in b and has even b's$\}$	abaab	abba, λ	$(a^*ba^*b)(a^*ba^*b)^*$

5] Def^n. L is a regular language if it has a RE α s.t. $L(\alpha) = L$.

e.g. see above.

e.g. Is $L_p = \{w \mid w = w^R\}$ regular?

L_p Not regular.

e.g. $L = \{w \mid w = a^n b^n$ for $n \geq 1\}$ | $a^* b^*$

e.g. aaabbb

Not regular | $ab + aabb + aaabbb + \cdots$

6] Thrm: Regular languages are closed under

union, | $L_1 = L(\alpha_1)$ $L_2 = (\alpha_2)$

$\alpha_1 + \alpha_2$

union,
intersection,
complement
concatenation,
and star operations

$L_1 = L(\alpha_1)$ $L_2 = (\alpha_2)$
$\alpha_1 + \alpha_2$

$\alpha_1 \alpha_2$
$(\alpha_1)^*$

7] Thrm: any finite language is regular?

$L = \{ w_1, w_2, w_3, ----, w_n \}$