Languages

Sunday, April 10, 2022 9:41 PM

Recall: alphabet

star operation

$a^* = \{\lambda, a, aa, \ldots\}$

$\{a,b,c\}^* = \{\lambda, a, b, c, aa, ab, ba, \ldots\}$

12] The reverse of a strings:

s^R is the same string spelled backwards.

e.g.

$(abac)^R = caba$

13] Thrm. $\forall x, y \in \Sigma^*$

$$(x \cdot y)^R = y^R \cdot x^R$$

14] Languages:

a language L is a set of strings on Σ

i.e. $L \subseteq \Sigma^*$

e.g.

on $\Sigma = \{a, b\}$

$L_1 = \{a, aaa, abba\}$

$L_2 = \{aaa\}$

$L_3 = a^*$

$L_4 = \emptyset$ is the empty language

$L_5 = \Sigma^*$

15] Word: the strings in a language are called word

16] The string-function: $w \in \Sigma^*$

$w : \{1, 2, 3, \ldots, |w|\} \longrightarrow \Sigma$

$w(i)$ is the i^{th} symbol in w

charAt[i] $= w[i+1]$

e.g. $w = abbc$, $w(1) = a$

$w(2) = w(3) = b$

$w(4) = c$

17] The language of palindromes $L_p = \{w \mid w = w^R\}$

$L_p = \{\lambda, a, b, aa, bb, aba, bab, aaa, bbb, \ldots\}$

18] Note: we can define infinite languages using set notations:

e.g.
1. $L = \{w \in \Sigma^* \mid w$ has some properties$\}$
2. $L = L_1 \cup L_2$
3. $L = L_1 \cap L_2$
4. $L = L_1^*$ (see [19, 20])

19] Language concatenation $L_1 \cdot L_2 = \{w \mid w = x \cdot y, x \in L_1, y \in L_2\}$

Notation L^n is the set of all strings made of concatenating n strings from L.

20] The closure operation (Kleene star)

L^* is the set of all strings obtained by concatenating zero or more strings from L.

e.g. $\{ab, bb\}^* = \{\lambda, ab, bb, abab, abbb, bbab, ababab, abbbab, \ldots\}$

21] Notation L^+ is the set of all strings obtained by concatenating one or more strings from L

21] Notation L^+ is the set of all strings obtained by concatenating one or more strings from L.

i.e. $L^+ = L \cdot L^*$

e.g.

$\{a, bb\}^+ = \{a, aa, bb, abb, bba, aaa, aaaa, bbbb, aabb, bbaa, abba, \ldots\}$

Exer: $\{a, b, bb\}^* = \{\lambda, a, b, bb, bab,$

22] Thrm: for any two languages A and B

if $A \subseteq B$ then $A^* \subseteq B^*$

e.g. $A = \{\underline{a}, ab, \underline{b}, bba\}$ a language on $\Sigma = \{a, b\}$

Find A^*

Notice $\Sigma = \{a, b\} \subseteq A$

So, $\Sigma^* \subseteq A^*$ by Thrm [22]

since A^* is a language on Σ^*, $A^* \subseteq \Sigma^*$

$\therefore A^* = \Sigma^*$

8 eg lar xpression

Objective: to have a powerful notation that represent a language.

e.g. (he in th

$A = \{x \quad x = 2k+1, \text{ for some } k \in \mathbb{Z}\}$

$A = \{x \mid x - r \quad, \text{ (r some } k \in \mathbb{Z}\}$

$\cdot := \{x \mid x = 2^k \quad, \text{ for so } \quad k \in \mathbb{Z}\}$

1 Regu r Expression (RE) over an alphabet Σ is a string on the alphabet Δ, where $\Delta = \Sigma \cup \{*, +, (,), \emptyset\}$, such that

1. $\emptyset$ and $\forall a \in \Sigma$ are the basic REs.
2. if α is RE, then so is α^* (star)
3. if α and β are REs, then so is $\alpha\beta$ (concatenation)
4. if α and β are REs, then so is $\alpha+\beta$ (union)
5. Note: nothing else is RE, unless it is made of the above rules with parantheris.

e.g. let $\Sigma = \{a, b\}$

by rule 1

$\alpha_1 = a$ is a basic RE for the language $\{a\}$

$\alpha_2 = b$ " " " " $\{b\}$

$\alpha_3 = \emptyset$ " " " " $\{\ \}$

by rule 2

a^* for $L = \{\lambda, a, aa, aaa, \dots\}$

b^* for $L = \{\lambda, b, bb, \dots\}$

$\emptyset^*$ for $L = \{\lambda\}$

by rule 3

aa for $L = \{aa\}$

$(aa)^*$ for $L = \{\lambda, aa, aaaa, \dots\}$

$a^* b^*$ for $L = \{\lambda, a, b, aa, ab, bb, aaa, \dots\}$

by rule 4

$$(a+b) = \{a, b\}$$
$$(a+b)^* = \Sigma^*$$

$a(a+b)^* bb$ → all string start in a and
end end in bb.