Alphabet
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Objective: to introduce the topic of the theory of Computation, which is the mathematical study of computing machines and their capabilities.

Lecture Notes on Theory of Computation (LNTC)

§1. Alphabet and languages

language $\longrightarrow$ words $\longrightarrow$ Letters

each letter $\in$ Alphabet

each word $\in$ Language

e.g. English Alphabet (A) // $A = \{a, b, c, \dots, z\}$

English language (L)

$a, p, l, e \in A$

$\hookrightarrow$ make: "apple" $\in L$ $\Rightarrow$ apple is a word in L

$\hookrightarrow$ aaea $\notin L$ $\Rightarrow$ aaea not a word.

1] An alphabet is a finite set of symbols, denoted by Σ

e.g.
$\Sigma = \{a, b, c\}$ // has 3 symbols
$\Sigma = \{0, 1\}$ // has 2 symbols.
$\Sigma = \{a, <, 3, \#, \dot{1}, @, 6\}$ // any thing can be a symbol.
$\Sigma = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, \dots\}$ $\leftarrow$ not valid.
$\Sigma = \{a\}$ // valid
$\Sigma = \{\} = \emptyset$ // valid, it could be empty;

2] A string on Σ is a finite sequence of symbols from Σ.
e.g.
for $\Sigma = \{a, b, c\}$; aabba [illegible]

e.g.

for $\Sigma = \{a, b, c\}$: aabbc, ac, cc are strings

$\Sigma = \{a\}$: aa, aaaa, a are strings

3] The length of a string s,

$$|s| = \text{number of symbols in } s.$$

e.g.

$s = 1100 \implies |s| = 4$

$s = aaa \implies |s| = 3$

4] The empty string (or null) is a string of length zero

- it is defined on every alphabet.
- denoted by λ

$$|\lambda| = 0$$

5] Note: any object can be a symbol. However, we use common chararcters like $\{0,1\}$, $\{a, b\} \leftarrow$ our default.

6] Concatenation: of two strings x and y on some Σ is

$$x \cdot y \text{ or simply } xy \quad \text{(Juxtaposition)}$$

e.g. $x = abb$, $y = aaa$

$x \cdot y = abbaaa$

$yx = aaaabb$

$xx = abbabb$

$x \cdot \lambda = x$

$\lambda \cdot x = x$

7] Notation: $s^n = \underbrace{s \cdot s \cdot s \cdot \dots \cdot s}_{n\text{-times}}$

$s^0 = \lambda$

Recursively, $s^n = \begin{cases} \lambda & \text{if } n = 0 \\ s \cdot s^{n-1} & \text{if } n > 0 \end{cases}$

e.g $x = abb$

$x^4 = abb\,abb\,abb\,abb$

$(aa)^3 = aa\,aa\,aa$

$a^3 = aaa$

8] In general,

let A be a set of symbols, then

$A^n = \{ s \mid s$ is a string on A of length $n \}$

e.g.
$A = \{b, c\}$

$A^3 = \{ bbb, bbc, bcb, bcc, cbb, cbc, ccb, ccc, \dots \}$

e.g.

$\{a, b, c\}^2 = \{ aa, ab, ac, ba, bb, bc, ca, cb, cc \}$

e.g. Σ^n is the set of all strings of length n on Σ.

9] Kleene star operation: let $a \in \Sigma$, then

a^* is the set of all strings made of a repeated zero or more time

$a^* = \{ s \mid s = a^n$ for all $n \geq 0 \}$

$= \{ \lambda, a, aa, aaa, \dots \}$

10] In general the Kleene star oprator can be applied on any set of symbols A,

$A^* \quad \{ s \quad s$ is a s ing on $- \}$

e.g ① Σ^* is t set of ll possible string on Σ.

② $\{a, c\}^* = \lambda, a, c, aa, ac, ca, cc, aaa, \dots \}$

Exer:

$a^* \cup b^* = \{a, b\}^*$ No, for $aba \in \{a, b\}^*$ but not in LHS

$a^* \cap b^* = \emptyset$ False, for $\lambda \in$ LHS but not in $\emptyset$

11] Substrings:

string y is a substring of w iff $\exists\, x, z \in \Sigma^*$ s.t.

$$w = xyz$$

y is called prefix of w if x is null.
y is called suffix of w if z is null.

e.g. aa is substring of $baaab$ for $x = b, z = ab$

ab is suffix of $baaab$ for $x = baa, z = \lambda$