Topological Sorting

Sunday, April 3, 2022 9:43 PM

Missing:
#5, 28
+B # 34, 9

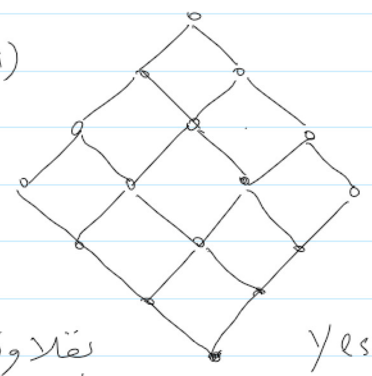
Objectives:

1. To learn more about lattices.
   Can an infinite poset be a lattice?
2. To study an application of partial order:
   (topological sorting)

Recall: Lattices
Posets

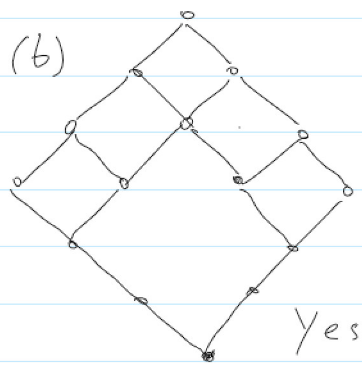
1] e.g. on lattices

(a)



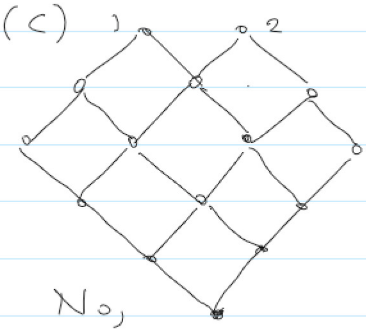
Yes

(b)



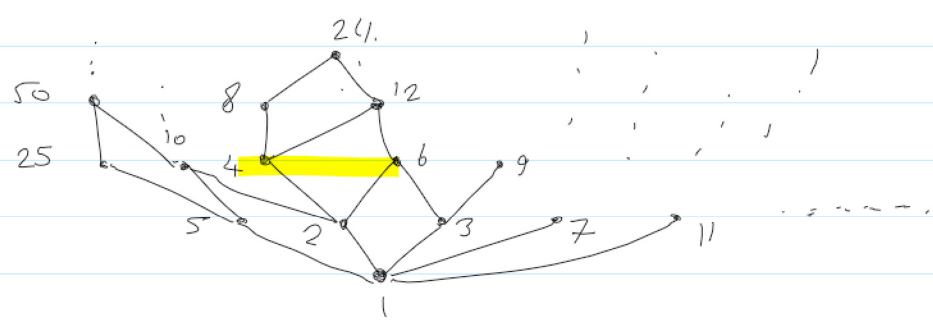
Yes

(c)



No,
for {1,2} has no LUB

2] e.g. $(\mathbb{Z}^+, |)$ is it lattice?

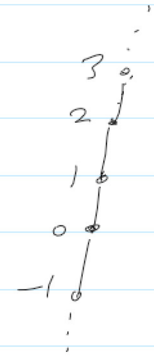


Yes, $\forall x, y \in \mathbb{Z}^+$
the greatest lowerbound (x,y) is $\gcd(x,y)$
the least upperbound (x,y) is $\operatorname{lcm}(x,y)$

3]

3] e.g. Is $(\mathbb{Z}, \leq)$ a lattice?

yes, $\forall x, y \in \mathbb{Z}$, $GLB(x,y) = \min(x,y)$
$LUB(x,y) = \max(x,y)$



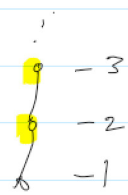
4] exer. Is $(\mathbb{Z}^-, \geqslant)$ a lattice?

No, not reflexive

+B#34

5] exer. Is $(\mathbb{Z}^-, \geqslant)$ a lattice?

Yes, $\forall x, y \in \mathbb{Z}^-$, $GLB(x,y) = \max(x,y)$
$LUB(x,y) = \min(x,y)$



+B#9

6] Topological Sorting:

objective: • to find a compatible total orderin on a poset
• to sort the tasks of a given project.

7] Lemma 1: every finite nonempty poset $(S, \preceq)$ has a minimal element.

8] Algorith: Topological Sort (S: finite poset)

1. $k = 1$
2. while $(S \neq \emptyset)$ do
2.1 { a_k = minimal_elt(S)

2. while (...) do

2.1 { a_k = minimal_elt (S)

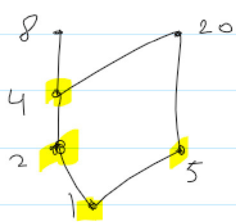
2.2. $S = S - \{a_k\}$

2.3. $k = k + 1$

}

3. output : $a_1, a_2, \ldots, a_n$ as a total ordering compatible with $(S, \preceq)$

9] e.g. Find a compatible total ordering for the poset $(\{1, 2, 4, 5, 8, 20\}, |)$



minimal →

output (compatible total ordering)

1 , 5 , 2 , 4 , 8 , 20

End of Ch 9.