Recall: Hasse Diagram

e.g. R on $A = \{x \in \mathbb{Z} \mid 1 \leqslant x \leq 10\}$
$x R y$ iff $x \mid y$

1] Def^n. elements with extremal properties in a poset $(S, \leq)$

1. a is the greatest element if $b \leq a$ $\forall b \in S$
2. a is the least element if $a \leq b$ $\forall b \in S$
3. a is maximal element if there is no $b \in S$, s.t. $a < b$
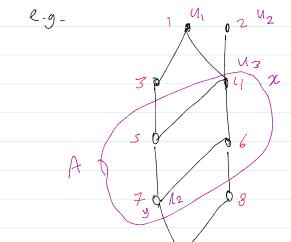4. a is minimal element if there is no $b \in S$, s.t. $b < a$

For $A \subseteq S$:

1. $u \in S$ is an upper bound of A if $a \leq u$ $\forall a \in A$
2. $l \in S$ is a lower bound of A if $l \leq a$ $\forall a \in A$
3. x is the least upper bound of A if $x \leq u_i$ $\forall$ U.B. u_i of A
4. y is the greatest lower bound of A if $l_i \leq y$ $\forall$ L.B. l_i of A

e.g.

Find all special elements. (03/30)
Greatest elts: none
least elts: 9
max elts: 1, 2
min elts: 9
UB of A: 4, 1, 2
LUB of A: $x = 4$
LB of A: 9, 7
GLB of A: $y = 7$

$9 \quad l_1$

LB of A: $9, 7$
GLB of A: $y=7$