Partial Order

Wednesday, March 23, 2022 12:46 PM

Recall: equivalence relation

ER = reflexive, symmetric, transitive

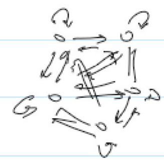
Missing:

\# 5, 6, 11, 12, 13, 16

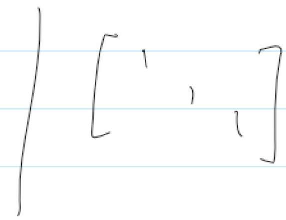
e.g. Let $|A| = 5$, and R is an ER on A with one class

① find $|R|$

$\therefore |R| = 5 \times 5 = 25$



$\forall a \in A$



② how many relation are there on A?

2^{25}

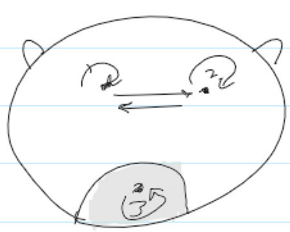
③ How many ER are there on A if $|A| = 3$?

HW

1] e.g. Let $A = \{1, 2, 3\}$

$R = \{(1,2), (1,1), (2,2), (2,1), (3,3)\}$,

is R an ER? find the partitions if so.

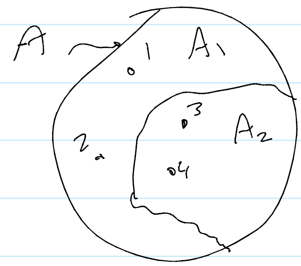


2] Thrm: Given a partition $\{A_i \mid i \in I\}$ of a set A, each set A_i is an equivalence class for the relation induced by this Partition.

e.g.

find an ER, R, on A

e.g.



find an ER, R, on A

$R = \{(1,1), (1,2), (2,1), (2,2),$
$(3,3), (3,4), (4,3), (4,4)\}$

end of § 9.5

§ 9.6 Partial Order

Objective: to define order of elements in a set based on some relation.

e.g.

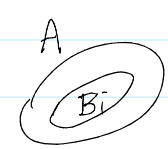
list your brothers names (R: older than)
take courses in a school (R: prerequisite)

3] Def^n. a relation R on A is a partial order if R is reflexive, anti-symmetric, transitive.
Here, (A, R) is called a partially ordered set or poset.

e.g ① $(\mathbb{Z}, \geq)$; $x R y$ iff $x \geq y$

e.g. ② $(P(A), \subseteq)$ for any set A.

e.g ③ $(\{2, 3, 4, 6, 8, 12\}, |)$



$B_i \subseteq A$

Power set of A
$P(A) = \{X \mid X \subseteq A\}$

4] Notation: $x \preceq y$ denotes $x R y$ in (A, R)
or $x \prec y$ if $x R y$ and $x \neq y$.

in e.g ③ $\quad 2 \prec 4$

5] Def^n. in a Poset $(S, \preccurlyeq)$, a and b are said to be comparable if $a \preccurlyeq b$ or $b \preccurlyeq a$, otherwise, a and b are incomparable.

in e.g ③ $\quad$ 2 and 8 are comparable,
while 6 and 8 are incomparable.

6] Def^n. (Total order)
if all elements in a poset $(S, \preccurlyeq)$ are comparable, then S is called a totally ordered set, and $\preccurlyeq$ is called a total order, (or a linear order)

e.g. $(\mathbb{Z}, \leq)$

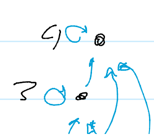
7] Lexicographic order (dictionary order)
for Poset $(A_1 \times A_2 \times A_3 \times \cdots \times A_n, \preccurlyeq)$, where $(A_i, \preccurlyeq)$ is a poset.
then $(a_1, a_2, \cdots, a_n) \prec (b_1, b_2, \cdots, b_n)$ if
$(a_1 \prec b_1)$ or $(a_1 = b_1, \cdots, a_i = b_i$, and $a_{i+1} \prec b_{i+1})$,
we add equality for the same element to set $\preccurlyeq$

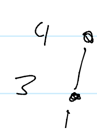
e.g. $(1, 2, \underline{3}, 5) \prec (1, 2, \underline{4}, 2)$

8] Hasse Diagrams

e.g. $(\{1, 2, 3, 4\}, \leq)$

1. make all arrows upwards
2. remove loops



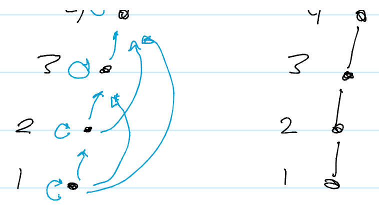


1. make all arrows upward

2. remove loops

3. remove arrows due to transitivity

4. remove arrows' heads



e.g.

$(\{2, 3, 4, 6, 8, 12\}, \mid)$