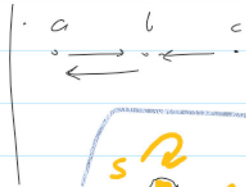
Equivalence Relations
Monday, March 21, 2022 12:41 PM

Recall: relation properties
reflexive, symmetric, transitive...

Objective: to study equivalence relations and classes.

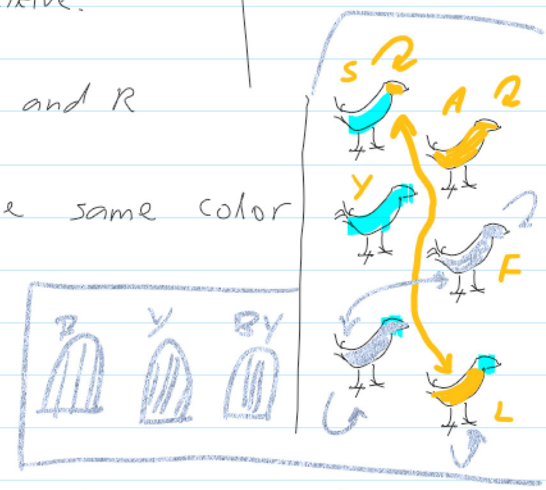
Missing
#2, 6, 34.
Ex #26 on 3/16

1] Def^n: R is called an equivalence relation (ER) if it is reflexive, symmetric, and transitive.



e.g. let A be a set of all birds, and R is a relation on A s.t.
$x R y$ iff x and y have the same color

e.g. let $m \in \mathbb{Z}^+$
R is a relation on $\mathbb{Z}$
s.t. $x R y$ iff $x \equiv y \pmod m$



2] Let R be an ER on A, then $\forall\ b \in A$ the set of all elements related to b is called the equivalence class of b, denoted by

$$[b]_R = \{ x \in A \mid x R b \}$$

if $y \in [b]$, then y is a representative of this equivalence class. $[y] = [b]$

3] Equivalence classes and Partitions

let R be an ER on A, then

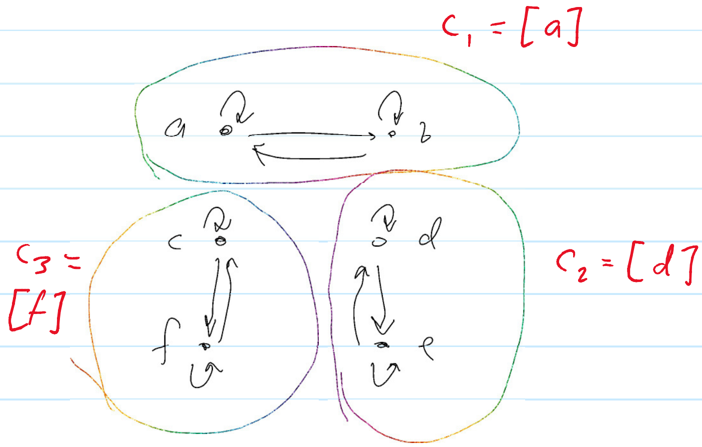
① if $aRb \Leftrightarrow [a] = [b]$

② if $a \not R b \Rightarrow [a] \cap [b] = \emptyset$

③ $\bigcup_{x \in A} [x] = A$

4] e.g. $A = \{a, b, c, d, e, f\}$

$$\begin{array}{c|cccccc} & a & b & c & d & e & f \\ \hline a & 1 & 1 & 0 & 0 & 0 & 0 \\ & 1 & 1 & 0 & 0 & 0 & 0 \\ c & 0 & 0 & 1 & 0 & 0 & 1 \\ d & 0 & 0 & 0 & 1 & 1 & 0 \\ e & 0 & 0 & 0 & 1 & 1 & 0 \\ & 0 & 0 & 1 & 0 & 0 & 1 \end{array}$$



R makes 3 partition of A:

$A_1 = \{a, b\}$
$A_2 = \{c, f\}$
$A_3 = \{d, e\}$

$A_1 \cup A_2 \cup A_3 = A$
$A_i \cap A_j = \emptyset$ for $i \neq j$

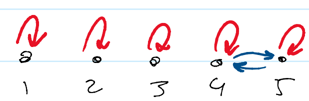
5] R is on $\mathbb{Z}$ defined: $a R b$ iff $a \equiv b \pmod 5$

$[0] = \{0, 5, 10, 15, \ldots, -5, -10, \ldots\}$
$[1] = \{\ldots, -9, -4, 1, 6, 11, 16, \ldots\}$
$[2] = \{\ldots, -13, -8, -3, 2, 7, 12, 17, \ldots\}$
$[3] = \{\ldots, -7, -2, 3, 8, 13, 18, \ldots\}$
$[4] = \{\ldots, -6, -1, 4, 9, 14, 19, \ldots\}$

$[0] \cup [1] \cup [2] \cup [3] \cup [4] = \mathbb{Z}$

6] exer. What is the number of classes if

① $|A| = 5$ and $|R| = 7$?
: 4

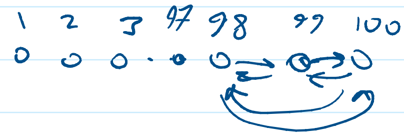


② $|A| = 5$ and $|R| = 9$?

(3) $|A| = 5$ and $|R| = 25$?

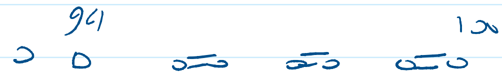
(4) $|A| = 5$ and $|R| = 24$?

(5) $|A| = 100$ and $|R| = 106$?



either 98 or 97 classes

$97 + 1 = 98$



$94 + 3 = 97$