Relation Digraphs

Monday, March 7, 2022 12:44 PM

Recall : Representing relation
- Sets
- Matrix-representation
- Digraph (Today)

Missing: #1, 30

#11 attended 03.02
Ex #3 on 03.02

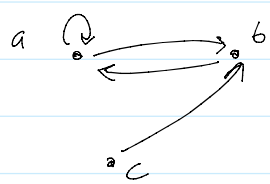
OBJECTIVE:
To represent relations and visualize their properties on graphs.

1] Digraph: directed graph.

$R = (V, E)$ V is a set vertices (nodes)
E is a set of edges.

R is a relation on V.
$E = \{(x,y) \in R\}$

2] e.g. $A = \{a, b, c\}$
$R = \{(a,a), (a,b), (b,a), (c,b)\}$

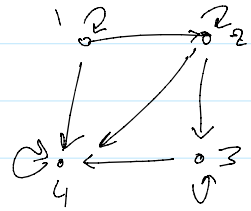


$x \to y$ iff $(x,y) \in R$

3] exer. Determine if R is ref, sym, trans, anti-sym, ...

R is:
Not reflexive for $b \not R b$
Not Symmetric for $c R b$ but $b \not R c$
Not Transitive for $c R b$ and $b R a$ but $c \not R a$
Not anti-symmetric for $a R b$ and $b R a$
Not asymmetric for $a R a$
Not irreflexive for $a R a$

4] Exer:

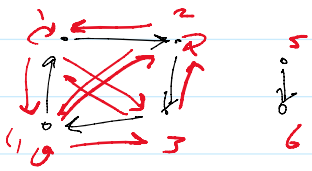
	ref	sym	trans	anti-sym	asym	irref
R_1 	yes	No	No for $1 R 3$	Yes	No	No
R_2 $1 \rightleftarrows 2$	No	Yes	No for $1 R 1$	No	No	Yes
$R_3 = \{(1,1)\}$ $1 \quad 2$	yes	yes	yes	yes	No	No
R_4 $1 \quad 2$ $R_4 = \{\ \} = \emptyset$	No	yes	yes	yes	yes	yes

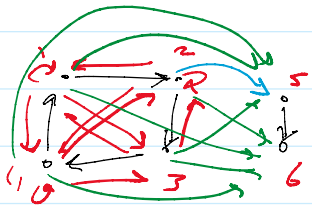
5] Note: From the digraph

reflexive? $\Longrightarrow$ all loops

symmetric? $\Longrightarrow$ either pair of edges or nothing

transitive? $\Longrightarrow$





anti-sym? $\Longrightarrow$ no $x \rightleftarrows y$

asym? $\Longrightarrow$ no $x \rightleftarrows y$ and no loops

irreflexive? $\Longrightarrow$ no loops

6] Exer:

Let R be a relation

$x R y$ iff:

	ref	sym	trans	anti-sym	asym	irref
+						

Let R be a relation xRy iff:	Ref	Sym	Trans	Anti-sym	Asym	Irref
① $x \le y$ on $\mathbb{Z}^+$	Yes	No	Yes	Yes	No	No
② $x < 2y$ on $\mathbb{Z}^+$	Yes	No 1R4, 4R1	No 7R4, 4R3, but 7R3	No 4R3, 3R4	No 1R1	No 1R1
③ $x = y$ on $\mathbb{Z}$	Yes	Yes	Yes	Yes	No 1R1	No 1R1
④ $x \equiv y \pmod{11}$ on $\mathbb{Z}$	Yes	Yes	Yes	No $1 \equiv 12$ $12 \equiv 1$	No $1 \equiv 1$	No $1 \equiv 1$