Representing Relations

Wednesday, March 2, 2022 12:37 PM

Recall: Relations properties
- reflexive
- symmetric
- transitive
- anti-symmetric: $(xRy) \wedge (yRx) \longrightarrow x = y$
  e.g. $\leq$ on $\mathbb{R}$

Missing:
3, 11, 25

1] Def^n. Let R be a relation on A, then

① R is asymmetric: if $\forall x, y \in A$, $xRy \longrightarrow y \not{R} x$

e.g $<$ on $\mathbb{R}$

② R is irreflixive: if $\forall x \in A$, $x \not{R} x$

e.g. $<$ on $\mathbb{R}$

2] Relation operations:
Objective: to have notation for expressing complicated relations in terms of other simple relations

Note: relations are sets $\Longrightarrow$ all set operations can be applied on relations.

e.g. $R_1 \cup R_2 = \{(x,y) \mid x R_1 y \vee x R_2 y\}$

$R_1 \cap R_2 = \{(x,y) \mid x R_1 y \wedge x R_2 y\}$

$R_1 - R_2 = \{(x,y) \mid x R_1 y \wedge x \not{R}_2 y\}$

$R_1 \oplus R_2 = (R_1 \cup R_2) - (R_1 \cap R_2)$

$\overline{R_1} = \{(x,y) \mid x \not{R}_1 y\}$ for a given universal set.

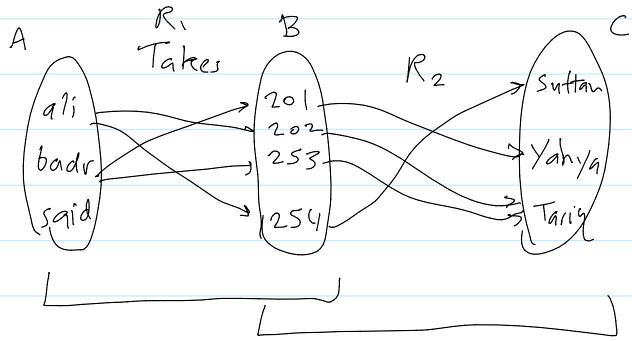
3] Def^n. Composition of two relations R_2 and R_1 is

$R_2 \circ R_1$ where R_1 is from A to B
and R_2 is from B to C

defined by $\underbrace{R_2 \circ R_1(x)}$ $= R_2(R_1(x))$ $\forall\, x \in A$

apply R_2 after R_1 on x

e.g. R_1 = Takes from A to B
R_2 = Taught_by from B to C
Then $R_2 \circ R_1$ = Student_of



$R_2 \circ R_1(ali) = \{Sultan, Tariq\}$

e.g. Parent $\circ$ Parent = grandparent

4] Def^n. let R be a set on A, then

$$R^n = \begin{cases} R & \text{if } n = 1 \\ R^{n-1} \circ R & \text{if } n > 1 \end{cases}$$

e.g. $R^2 = R^1 \circ R = R \circ R$

$R^3 = R^2 \circ R = (R \circ R) \circ R$

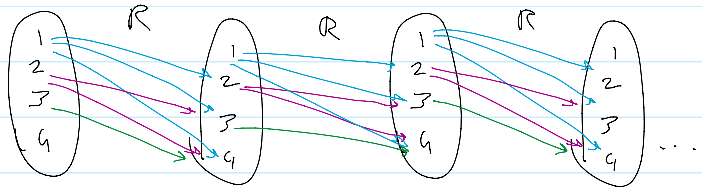
5] Thrm : R is transitive iff $R^n \subseteq R$

e.g. let R be a relation on $\{1, 2, 3, 4\}$
defined by $x R y$ iff $x < y$

$R^2 = \{(1,3), (1,3), (2,4)\}$

$R^3 = \{(1,4)\}$

$R^4 = \{\ \} = \emptyset$



§ 9.3. Representing Relations

6] Matrix of R (from A to B) M_R

M_R is a $|A| \times |B|$ matrix $[a_{ij}]$

$$a_{ij} = \begin{cases} 1 & \text{if } x_i R x_j \\ 0 & \text{otherwise} \end{cases}$$

e.g. $A = \{1, 2, 3\}$, $B = \{1, 2\}$

$R = \{(2,1), (3,1), (3,2)\}$

$$M_R = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cc} 1 & 2 \end{array} \\ \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 1 \end{bmatrix} \end{array}$$

e.g. $\leq$ on $A = \{1, 2, 3, 4\}$

$$M = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

7] Relation Properties on M_R

reflexive Symmetric anti-symmetric