

Relations

Monday, February 28, 2022 12:45 PM

Missing

2, 5, 32

OBJECTIVES:

- To study mathematical relations and their properties
- To learn how they relates to Computer Science.

§ 9. Relations:

"relation between objects"

e.g. $x < y$ on \mathbb{R}

Ahmad is a brother of Bander

§ 9.1. Properties of Relations

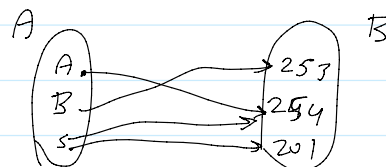
1] Defⁿ. a (binary) relation R from A to B is a subset of $A \times B$, where $A \times B = \{ (x, y) \mid x \in A \text{ and } y \in B \}$

e.g. 1: $A = \{ \text{Ali, Bander, Said} \}$

$B = \{ 253, 254, 201 \}$

Relation: Takes = $\{ (\text{Ali}, 254), (\text{Bander}, 253), (\text{Said}, 254), (\text{Said}, 201) \}$

Brother	A	B	S
A			✓
B	✓		
S			



2] Notation:

$a R b$ iff $(a, b) \in R$

e.g.

$(\text{Ali}, 254) \in \text{Takes}$

$\Leftrightarrow \text{Ali Takes } 254$

$ A \times B $		A	B	S
= 9	253		✓	
	254	✓		✓
	201			✓
$ \text{Takes} = 4$				

$$\underline{\underline{5 < 15}} \Rightarrow (5, 15) \in <$$

Exer: let $<$ be a relation, on $A = \{ 1, 2, \dots, 10 \}$

Find $|<|$

$$\begin{aligned} \text{Sol. } \quad 9 + 8 + \dots + 1 + 0 &= \frac{9+10}{2} \times 10 = 45 \\ &= \frac{9+1}{2} \times 9 = 45 \end{aligned}$$

3] Defⁿ. an n -ary relation is a subset of

$$A_1 \times A_2 \times \dots \times A_n$$

* Note: (binary) is assumed by default

e.g. 2. $C = \{ \text{Sultan, Tariq, Yahya, } \dots \}$

Relation: $R = \{ (x, y, z) \mid x \text{ takes } y \text{ with } z \}$

$$R \subseteq A \times B \times C$$

e.g. $(\text{Ali}, 254, \text{Sultan}) \in R$

4] Defⁿ. Relation R is on a set A if $R \subseteq A \times A$

e.g. Brother relation on $A = \{ \text{Ali, Bander, Said} \}$

e.g. Harder on $B = \{ 253, 254, 201 \}$

$$\text{Harder} = \{ (254, 201), (253, 254), (252, 201) \}$$

knows on A

$$\text{e.g. knows} = \{ (A, A), (A, B), (B, A), (B, B), (S, A), (S, S) \}$$

5] Defⁿ. let R be a relation on A , then R is:

① reflexive: if $\forall x \in A, x R x$

e.g. knows and \leq

② Symmetric: if $\forall x, y \in A, xRy \rightarrow yRx$
e.g. Brother

Knows is not symmetric for sRa but a ~~R~~s
Harder is not symmetric for $253R254$
but $254R253$

③ transitive: if $\forall x, y, z \in A,$
 $(xRy) \wedge (yRz) \rightarrow xRz$
e.g. Harder

④ anti symmetric: if $\forall x, y \in A, (xRy) \wedge (yRx) \rightarrow x=y$
e.g. \leq on \mathbb{R}

Note:

$$(xRy) \wedge (yRx) \rightarrow x=y$$

by CP.

$$x \neq y \rightarrow \neg (xRy) \wedge (yRx)$$

$$x \neq y \rightarrow (xRy \vee yRx)$$

$$x \neq y \rightarrow (xRy \rightarrow yRx)$$

$$\left. \begin{array}{l} p \rightarrow q \\ \neg p \vee q \end{array} \right\}$$