Review

Wednesday, May 6, 2026 11:11 AM

Recall: Counting Recurrence Relations

Quiz 5

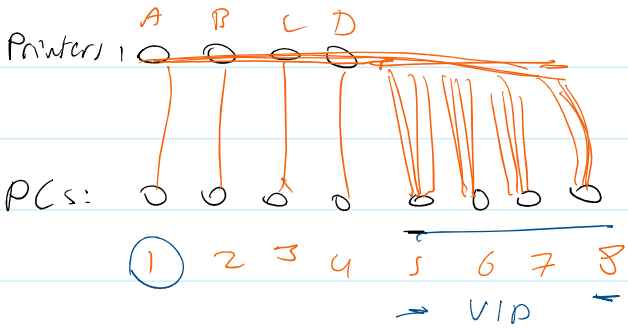
Here: Norah,

Here: 818, 826, 824

+B# 714, 728

+2B# 801

1] Exer. Given 8 PCs and 4 printers:
How many cables are needed?



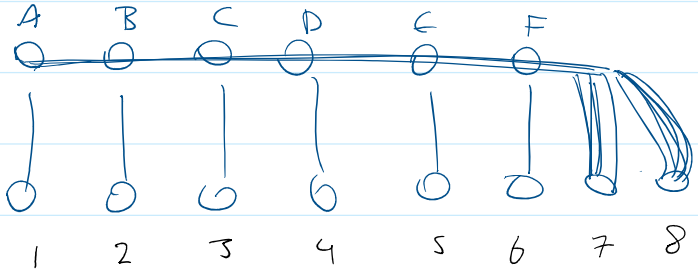
8 × 4 = 32 ✗

8 × 3 = 24 ✗

16 + 4 = 20 ✓

+B# 728

8 PCs + 6 printers



6 + 2×6 = 18

+B# 714

2] Exer. How many positive divisors does n have?

① $n = 17 \Longrightarrow$ 2 divisors

1	17

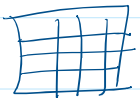
② $n = 16 = 2^4 \Longrightarrow$ 5 divisors

$2^i \mid 16$ for

i = 0	1	2	3	4	
1	2	4	8	16	16

③ $n = 6^3 = 2^3 \cdot 3^3 \Longrightarrow 2^i \cdot 3^j \mid 6^3$ for $i,j = 0 \ldots 3$

$4 \times 4 = 16$ divisors



④ $n = 18 = 2^1 \cdot 3^2 \Longrightarrow 2 \times 3 = 6$ divisors

$2^i \cdot 3^j \mid 16$ for $i = 0,1$ $j = 0,1,2$

	$j=0$	1	2
$i=0$	1	3	9
1	2	6	18

$\mid 18$

⑤ $n = 200$

$= 2^3 \times 5^2$

$\Longrightarrow 4 \times 3 = 12$ dir.

	0	1	2	3
0	1	2	4	8
1	5	10	20	40
2	25	50	100	200

$\mid 200$

3] Exer.

① Smallest n that has 8 divisors

$8 = \longrightarrow 2^7 = 128$

$= 2 \times 4$

$2^3 \cdot 3^1 = (24)$

$2^1 \cdot 3^1 \cdot 5^1 = 30$

$2 \cdot 2 \cdot 2$

$i=$	0	1	2	3
$j=0$	1	2	4	8
1	3	6	12	24

$\mid 24$

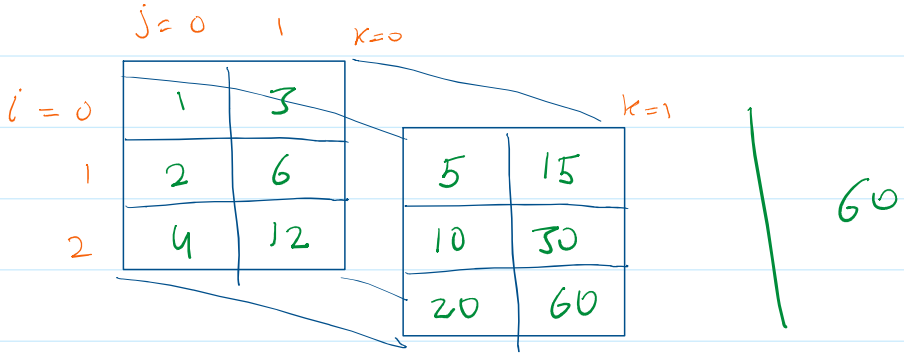
② Smallest n that has 12 divisors

$12 = \longrightarrow 12 \Longrightarrow n = 2^{11} = 2048$

$6 \times 2 \Longrightarrow n = 2^5 \cdot 3^1 = 96$

$4 \times 3 \Longrightarrow n = 2^3 \cdot 3^2 = 72$

$3 \cdot 2 \cdot 2 \Longrightarrow n = 2^2 \cdot 3^1 \cdot 5^1 = 60$



③ n has 14 divisors

$14 \longrightarrow 7 \times 2$

$\Rightarrow 2^6 \cdot 3^1 = 64 \times 3 = 192$

+2B#801

Quiz 5

F13

① $17^{20} \pmod{19}$

mod 18

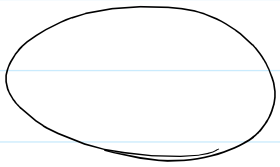
$\equiv (-2)^2 \equiv 4$

FLT $a^{p-1} \equiv 1 \pmod{p}$

② 82 people $\longrightarrow$ day of the week

$$\left\lceil \frac{82}{7} \right\rceil = 12$$

③ 6 faculty · 2 students on circle



7!

④ Stamps $\forall n \geq 26$

4 and 7 ✓ $\Rightarrow$ $20 = 5 \times 4$

$21 = 3 \times 7$

$22 = 14 + 8$

$23 = 16 + 7$

⑤ $P(1)$ $1^2 = \frac{1(2)(3)}{6}$

F14

#1. Car plates

Letters 2 or 3 digits 2 or 3

26 26 26 26 26 → 10 10 10 10 10

$(26^2 + 26^3)(10^2 + 10^3)$

#2. $P(1) \wedge (\forall k \; P(k) \rightarrow P(k+1)) \rightarrow P(n) \; \forall n$

#2. $P(1) \wedge (\forall k \; P(k) \rightarrow P(k+1)) \rightarrow P(n) \; \forall n$

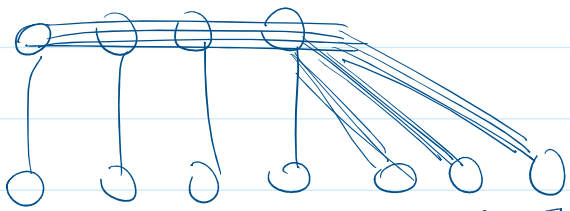
induction

#3. $12x \equiv 19 \pmod{23}$

$(2 \times 12)x \equiv (-4) \times 2$

$(1)x \equiv -8 \equiv 15 \pmod{23}$

#4.



$4 + 3 \times 4 = 16$

#5 $\forall n \geq 20$

4 and 7 ✓

$20 = 4 \times 5$

$21 = 7 \times 3$

$22 = 14 + 8$

$23 = 16 + 7$

+4 → 24