Permutations and Combinations

Wednesday, April 22, 2026 11:26 AM

Recall: Product rule

on Quiz 4

+B #810

§ 6.3 Permutations and Combinations

1] Permutation: an ordered arrangment of objects

$P(n,r)$ = the number of ways to list r out of n objects $(r \le n)$

$= n(n-1)(n-2)\cdots(n-r+1)$

$= \dfrac{n!}{(n-r)!}$

$C = \{R, B, Y, G, W, H\}$

$\boxed{6}\ \boxed{5}\ \boxed{4}\ \boxed{3} = \dfrac{6!}{2!}$

$n = 6, \quad r = 4$

$n=7 \quad \{M\ F\ W\ Z\ S\ N\ R\}$

$r=4 \quad \underline{7}\ \underline{6}\ \underline{5}\ \underline{4}$

2) e.g.

We have 34 students, in how many ways can 4 people line up for a photo.

Sol^n $\quad P(34,4) = \dfrac{34!}{30!}$

3] Combination: an unordered selection of objects.

$C(n,r)$ = number of ways to choose r out of n objects.

$= \dfrac{n(n-1)\cdots(n-r+1)}{r!} = \dfrac{n!}{(n-r)!\,r!}$

$\{H, L, R, G, Z\}$

$\underline{H}\ \underline{L}\ \underline{R}$

$P(5,3)/3!$

M F W Z S N R

4] Notation:

$$\binom{n}{r} = C(n,r)$$

R N Z
Z N R

5] Note: $P(n,n) = n!$

$$P(n,r) = \binom{n}{r} \cdot r!$$

$$\binom{n}{r} = \binom{n}{n-r}$$

e.g. $$\binom{10}{7} = \frac{10!}{7!\,3!} = \frac{10\cdot 9\cdot 8\cdot \cancel{7}\cdot \cancel{6}\cdot \cancel{5}\cdot \cancel{4}}{\cancel{7}\cdot \cancel{6}\cdot \cancel{5}\cdot \cancel{4}\cdot 3\cdot 2\cdot 1} = \frac{10\cdot 9\cdot 8}{3!}$$

$$\binom{10}{3} = \frac{10!}{3!\,7!}$$

6] e.g. How many subsets of $A = \{M, F, W, Z, S, N, R\}$ are there of

① size 3? $\binom{7}{3}$

Sol^n. $$\binom{7}{3} = \frac{7\cdot 6\cdot 5}{3!} = 35$$

② size 3 contain R? $\{R, \_, \_\}$

Sol^n $$\binom{6}{2} = \frac{6\cdot 5}{2!} = 15$$

③ size 7?

Sol^n $$\binom{7}{7} = \frac{7!}{(7-7)!\,7!} = \frac{7!}{1\cdot 7!} = 1$$

7] exer. How many bit strings are there of length 8

① with exactly three 1's ?

Sol^n.

$\underset{\times}{1}\ \underset{\times}{1}\ \underset{\times}{1}$ _ _ _ _ _ $\equiv$ $\underset{\times}{0}$ _ $\underset{\times}{0}$ _ _ $\underset{\times}{0}\ \underset{\times}{0}\ \underset{\times}{0}$

$$\binom{8}{3} = \frac{8!}{3!\,5!}$$

+B #810 ②

② have at least three 1's ?

$$\underbrace{\left(\binom{8}{0}+\binom{8}{1}+\binom{8}{2}\right)}_{\text{excluded}} \ \binom{8}{3} + \binom{8}{4} + \dots + \binom{8}{8}$$

excluded

$$\binom{8}{0} = \frac{8!}{0!\,8!} = 1$$

J' $2^8 - \left(\binom{8}{0} + \binom{8}{1} + \binom{8}{2}\right)$ ✓

Z $\binom{8}{3} + \binom{8}{4} + \dots + \binom{8}{8}$ ✓