Missing:

700 # 4, ~~8~~,

here: 804, 10, 13, 14, 18, 25

800 # 20

here # 708

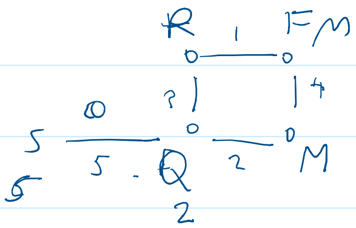
+B # 708, 724,

+B # 806, 809, 819

Recall: induction

see also: ppt p03

1] Gossip problem.



for $n=1$, 0 calls

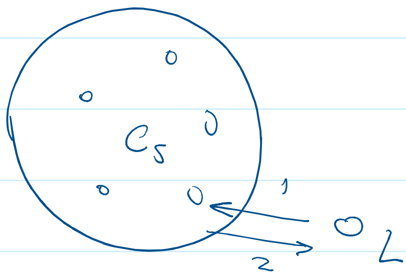
HW $n=2$, 1 call

⋮

4, 4 calls

+B # 819   5, 6 calls

6: , $1+6+1 = 8$ calls



C_{n+1}, $C_n + 2$ calls, $n > 4$

2] e.g. Robot can climb up 4 or 7 rungs at a time.

then it reach any rung $n \geq 18$

Proof:

Basis step: for $n = 18$, $7+7+4 = 18$

$n=19$, $3 \times 4 + 7 = 19$

$n=20$, $5 \times 4 = 20$

$n=21$, $3 \times 7 = 21$

Inductive step.

assume the robot can go to all rungs from 18 to $k \geq 21$,

then for $k+1$, it can go to $(k-3)$

then move 4 rungs up $= (k-3)+4 = k+1$

By strong induction, the robot can go to any $n \geq 18$

3] Prove that: $\overline{\bigcap_{i=1}^{n} A_i} = \bigcup_{i=1}^{n} \overline{A_i}$

Proof:

Basis step: for $n=2$

$$\overline{A_1 \cap A_2} = \overline{A_1} \cup \overline{A_2} \qquad \text{by DeMorgan's}$$

Inductive Step:

assume it is true for $n=k$,

$$\overline{\bigcap_{i=1}^{k} A_1} = \bigcup_{i=1}^{k} \overline{A_i}$$

W.t.s. $\overline{\bigcap_{i=1}^{k+1} A_1} = \bigcup_{i=1}^{k+1} \overline{A_i}$

$$LHS = \overline{A_1 \cap A_2 \cap \cdots A_k \cap A_{k+1}}$$

$$= \overline{\bigcap_{i=1}^{k} A_i \cap A_{k+1}}$$

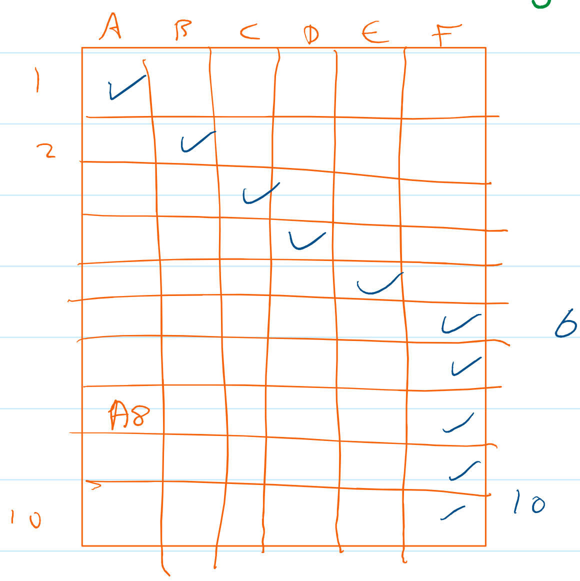
$$= \overline{\bigcap_{i=1}^{k} A_i} \cup \overline{A_{k+1}} \qquad \text{by DeMorgan's}$$

$$= \bigcup_{i=1}^{k} \overline{A_i} \cup \overline{A_{k+1}} \qquad \text{by assumption}$$

$$= \bigcup_{i=1}^{k+1} \overline{A_i} = RHS$$

$\therefore$ By mathematical induction, $\overline{\bigcap_{i=1}^{n} A_i} = \bigcup_{i=1}^{n} \overline{A_i}$ for all $n \geq 2$

§ 6.1 Basics of Counting



$6 \times 10 = 60$

4] Product rule:

if task A has n ways to do,
and task B has m ways,
then there are $n \cdot m$ ways to do A and B.

e.g. an ID of a letter and a digit, like A4

Soln. $\underline{26}$ $\underline{10}$ $26 \times 10 = 260$

5] Sum rule

if task A has n ways to do,
and task B has m ways,
then there are $n+m$ ways to do A or B.

e.g. to choose a char of lower case letter or digit

Soln. $26 + 10 = 36$

6] exer. ① Car plates of 3 letters and 4 digits

Soln.

$\underline{26}\ \underline{26}\ \underline{26}\ \ \underline{10}\ \underline{10}\ \underline{10}\ \underline{10}$ $\qquad 26^3 \cdot 10^4$

② VIP car plates have same digit 4 times

Soln.

$\underline{26}\ \underline{26}\ \underline{26}\ \ \underline{\quad 10 \quad}$ $\qquad 26^3 \cdot 10$

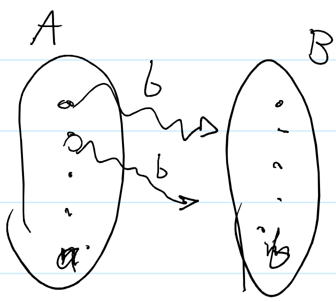
8] Exer. ① 5 people to choose 5 different cards.

Soln

$\underline{5}\ \underline{4}\ \underline{3}\ \underline{2}\ \underline{1}$ $\qquad 5!$

② # of function $f : A \longrightarrow B$

$|B|^{|A|}$ +B # 724



$\underbrace{b \cdot b \cdot b \cdot \ldots b}_{a \text{ times}}$

③ let $A = \{a, b, c, d, e, f\}$

How many subsets are there that contains b?

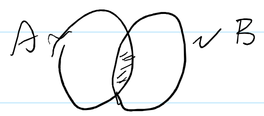
How many subsets are there that contains b?

Sol^n. $\underline{2}\ \underline{1}\ \underline{2}\ \underline{2}\ \underline{2}\ \underline{2}$ $\quad 2^5$

+B #809

9] Inclusion-exclusion principle (subtraction rule)

$$|A \cup B| = |A| + |B| - |A \cap B|$$



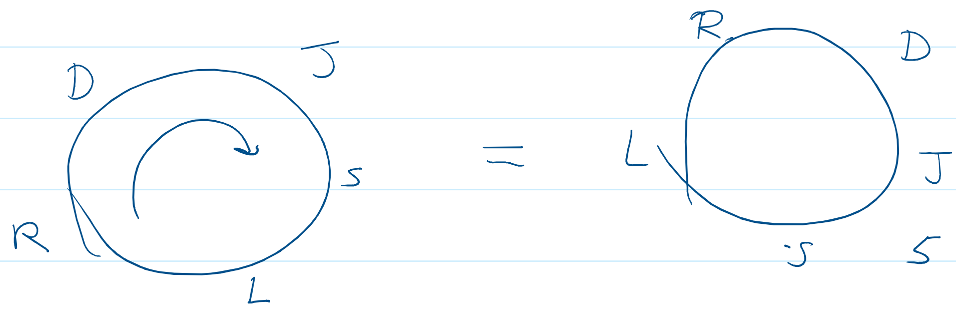
10] Division rule:

if task A has n ways, where each d ways are similar, then it has n/d ways

e.g. ① 5 people to line up for a photo

$\underline{J}\ \underline{S}\ \underline{L}\ \underline{R}\ \underline{D}$ $\quad 5!$

② 5 people to sit on a round table

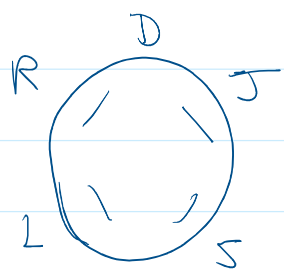


5 rotations

by division rule: $\frac{5!}{5}$

by a reference method $4!$

$(n-1)!$



1] Exer:

- How many 8-digit integers are there?

① with leading zeros allowed.

$$10^8$$

② ≥ 20000000 and end with 99

$\underline{8}\ \underline{10}\ \underline{10}\ \underline{}\ \dots\ \underline{}\ \underline{10}\ \underset{9}{\underline{1}}\ \underset{9}{\underline{1}}$

$8 \times 10^5 \times 1 \times 1 = 800000$

③ Start and end with 99

10^4 +B #806

④ Start or end with 55

2×10^6 ✗ $\qquad 10^6$ ✗ +B708

Sol^n $\quad \underline{5}\ \underline{5}\ \underline{}\ \underline{}\ \underline{}\ \underline{}\ \underline{}\ \underline{}$

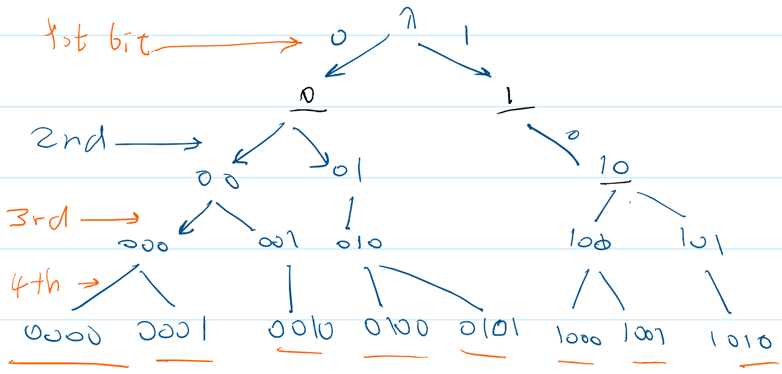
$5\ 5$

$$10^6 + 10^6 - 10^4$$

12] Tree diagram:

e.g. how many bit string are there of Length 4 without consecutive 1's

0010 ✓	1100 ✗
1010 ✓	0110 ✗

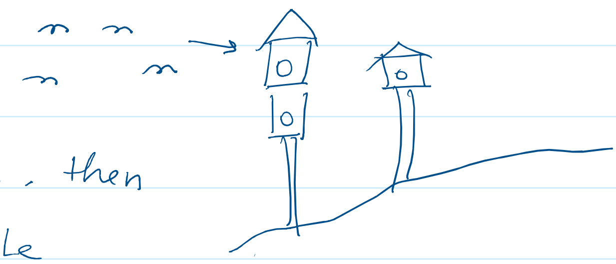


8 strings

§ 6.2 Pigeonhole Principle

13] Thm:

if n pigeons fly to m pigeonholes and $n > m$, then there is at least one hole with 2 or more pigeons.



e.g. $f: A \to B$,

if $|A| > |B|$, then f is not one-to-one

14] Thm:

In general, if n pigeons fly to m holes, then there is at least one hole with at least $\lceil n/m \rceil$ pigeons.

eg. How many of you (F14) are born on the same month. $n = 26$

at least $\left\lceil \frac{26}{12} \right\rceil = 3$ people

15] Exer: How many bit strings of 8 bits are there

① start in 00 or 11 ?

$$\underbrace{\underset{\to 1}{\overset{\to 0}{\_}}\ \underset{1}{\overset{0}{\_}}}_{2}\ \underbrace{\overset{2}{\_}\ \overset{2}{\_}\ \_\ \_\ \_\ \overset{2}{\_}}_{2^6}$$

2×2^6

② have exactly three 1's

_ _ _ _ _ _ _ _