§ 5.1 Mathematical Induction

See also: ppt slides on induction

Missing:
700 # 4, 11, 17, 23, 28
800 # 2, 3, , 13, 16,
+B # 811, 824, 825

1] Objective: To prove that $\forall n \; P(n)$
for all integers $n = 1, 2, \ldots$

2] Exer. Add n odd numbers

for $n = 2$:	$1 + 3$	$= 4$
$n = 3$	$1 + 3 + 5$	$= 9$
4	$1 + 3 + 5 + 7$	$= 16$
5	$1 + \ldots + 7 + 9$	$= 25$
6	$1 + \ldots + 9 + 11$	$= 36$
7	$1 + \ldots + 11 + 13$	$= 49$

3] Theorem:
The sum of the smallest n positive odd numbers is n^2

$$\forall n, \quad 1 + 3 + \cdots + (2n-1) = n^2$$

4] Proof by induction

Basis step: for $n = 1$
$LHS = 1$, $RHS = 1^2 = 1$

Inductive step:
Assume $1 + 3 + \cdots + (2k-1) = k^2$ inductive hypothesis
W.t.S: $1 + 3 + \ldots + (2k-1) + (2k+1) = (k+1)^2$

Assume $1 + 3 + \dots + (2k-1) = k$

W.t.s: $1 + 3 + \dots + (2k-1) + (2k+1) = (k+1)^2$

$$
\begin{aligned}
LHS &= 1 + 3 + \dots + (2k-1) + (2k+1) \\
&= k^2 + (2k+1) \quad \text{by assumption} \\
&= k^2 + 2k + 1 \\
&= (k+1)^2
\end{aligned}
$$

By mathematical induction first principle, the statement is true for all $n = 1, 2, \dots$