Divisibility

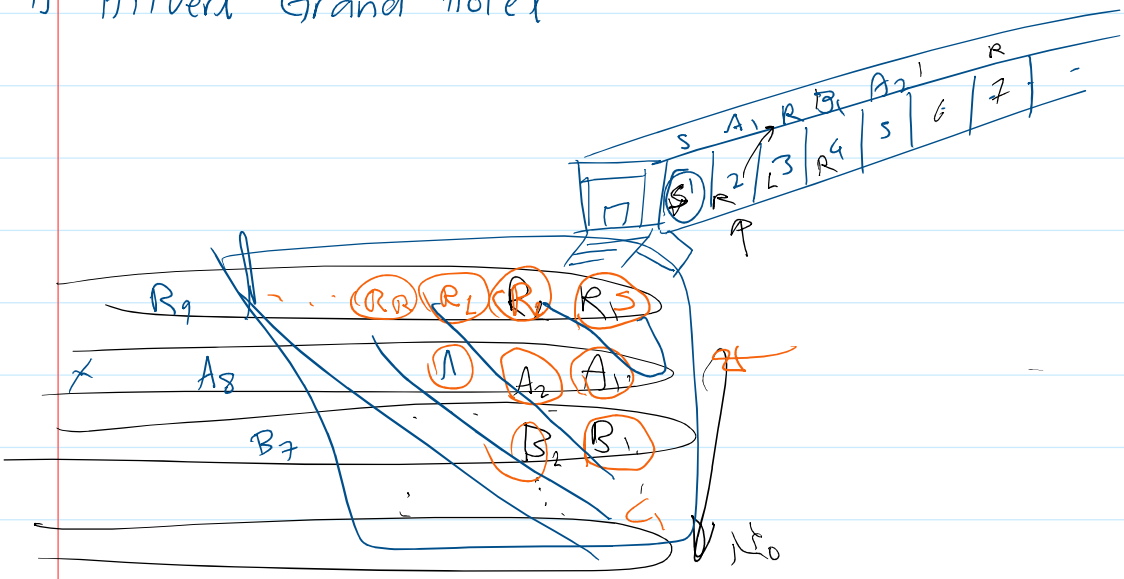
Monday, March 9, 2026 12:56 PM

Missing - 800 # 2, 3,

Recall : Countable set

$|\mathbb{Z}^+| = \aleph_0$

1] Hilbert Grand Hotel



$i \longrightarrow i + 20$

$i \longrightarrow 2i - 1$

§ 4.1 Divisibility and Modular Arithmetic

2] Division :

Def^n. Given two integers, a and b, with $b \neq 0$, we say b divides a if $\exists c \in \mathbb{Z}, a = bc$.

Notation : $b \mid a$ denotes b divides a

e.g. $3 \mid 12$, for $c = 4$

$3 \nmid 7$

3] Thm: let $a, b, c \in \mathbb{Z}$

① if $a \mid b$ and $b \mid c$, then $a \mid c$

② if $a \mid b$, then $a \mid bc$

③ if $a \mid b$ and $a \mid c$, then $a \mid (mb + nc)$ $\forall m, n \in \mathbb{Z}$

e.g. ③ $3 \mid 9$ and $3 \mid 15$

$3 \mid (2 \cdot 9 + 4 \cdot 15)$ for $3 \mid 78$

4] Note: $0 = 0 \times 5$

$\Rightarrow 5 \mid 0$ for $c = 0$

but $0 \mid 0$ for $c = 5$ ✗ this is wrong since $\mid$ is not defined on 0.