

Recall: Functions  
• bijection

online

1) Exer.  $f: \mathbb{R} \rightarrow \mathbb{R}$ ,  $f(x) = \lfloor \frac{-x}{2} \rfloor$

is this:

① one-to-one? No, for  $f(0.1) = f(0.2) = -1$

② onto? No, for  $y = \frac{1}{2}$  has no preimage

③ bijection No

2) Thm:  $\forall x \in \mathbb{R}, n \in \mathbb{Z}$

$$\lfloor x \rfloor = n \quad \text{iff} \quad n \leq x < n+1$$

$$\lceil x \rceil = n \quad \text{iff} \quad n-1 < x \leq n$$

$$x-1 < \lfloor x \rfloor \leq x \leq \lceil x \rceil < x+1$$

$$\lfloor -x \rfloor = -\lceil x \rceil$$

$$\lceil x+n \rceil = \lceil x \rceil + n$$

$$\lfloor x+n \rfloor = \lfloor x \rfloor + n$$

## § 2.5 Cardinality

3] Def<sup>n</sup>. A and B have same cardinality iff there is a bijection from A to B.

A is countable if A is finite or has same cardinality

as  $\mathbb{Z}^+$ , denoted by  $\aleph_0$  "aleph null" =  $|\mathbb{Z}^+|$

otherwise, it is uncountable.





10) Power set of  $\mathbb{Z}^+$

$P(\mathbb{Z}^+)$  is uncountable

Proof:

Given a list  $A_1, A_2, \dots$

Create  $X \subseteq \mathbb{Z}^+$

$$X = \{i \mid i \notin A_i\}$$

then  $X$  is not listed

A1.  $\emptyset$

A2.  $\{1\}$

A3.  $\{1, 2, 3\}$

A4. even

$\vdots$

$X$

	1	2	3	4	5	6	...
A1. $\emptyset$	0	0	0	0	0	0	...
A2. $\{1\}$	1	0	0	0	0	0	...
A3. $\{1, 2, 3\}$	1	1	1	0	...	...	...
A4. even	0	1	0	1	...	...	...
$\vdots$							
$X$	1	1	0	0	...	...	...

Worksheet PWD : " ? ? ? 4 5 6 7 8 "

↑  
watch the recording.