Sequences and Summations

Monday, March 2, 2026 11:28 AM

Missing

700 # 4, ~~8~~, 11,

800 # 3, ~~4~~, ~~7~~, ~~12~~, ~~15~~, ~~16~~, ~~22~~, ~~27~~, ~~26~~

Recall: Functions

1] Factorial:

$f: \mathbb{N} \to \mathbb{Z}^+$

$f(n) = n(n-1)(n-2)\cdots 1 \quad ; \quad f(0) = 1$

(denoted $n!$)

Thus, $n! = \begin{cases} 1 & \text{if } n = 0 \\ n(n-1)! & \text{if } n \geq 1 \end{cases}$

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§ 2.4 Sequences & Summations

2] Def^n. A sequence is a function $f: A \to B$; $A \subseteq \mathbb{Z}$

usually A is $\{0, 1, 2, \dots\}$ or $\{1, 2, \dots\}$

Notation: a_n denotes $f(n)$ is a term of the sequence

e.g. $\{a_n\}$ where $a_n = \frac{1}{n}$

is a sequence of the terms: $a_1, a_2, a_3, a_4, \dots$

Start with: $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots$

3] Def^n. geometric progression: is a sequence of the form

$$a, ar, ar^2, \dots, ar^n, \dots$$

with initial term a, and a common ratio r; $a, r \in \mathbb{R}$

e.g.

$1, 2, 4, 8, \dots$ initial term $= 1$, $r = 2$

4] Def^n. arithmetic progression: is a sequence of the form

$$a, \; a+d, \; a+2d, \; \ldots, \; a+nd, \ldots$$

with initial term is a, and d is the common differce

e.g.

$5, 7, 9, 11, \ldots$ initial term $= 5$, $d = 2$

5] Exer:

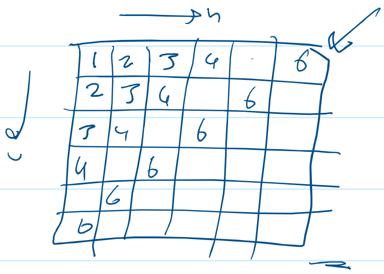
1. $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}$; Geometric, $a = 1$, $r = \frac{1}{2}$
2. $1, 8, 15, 22, \ldots$; Arithmetic, $a = 1$, $d = 7$
3. $1, -1, +1, -1, +1, \ldots$; Geometric, $a = 1$, $r = (-1)$

6] Integer Sequences: How to determine?

Look for: 1. runs of same number
2. terms obtained from previous term
3. cycles

7] Exer: Find the next term and the n^{th} term for $n = 1, 2, 3, \ldots$

1. $5, 11, 17, \underline{23}$ $\qquad a_n = 5 + 6(n-1)$
2. $1, 4, 9, \underline{16}$ $\qquad a_n = n^2$
3. $1, 1, 1, 2, 2, 2, 3, 3, 3, \underline{4}$ $\qquad a_n = \lceil \frac{n}{3} \rceil$
4. $1, 2, 2, 3, 3, 3, \underline{4}$ $\qquad a_n =$



HW

8] Summations :

Given a subsequence $a_m, a_{m+1}, a_{m+2}, \dots, a_n$

The sum of the terms is $\sum_{j=m}^{n} a_j = a_m + a_{m+1} + \dots + a_n$

Here

j is the index

m and n are the lower and upper limits.

e.g.

$$\sum_{j=1}^{100} j = 1+2+\dots+99+100 = \frac{1+100}{2} \times 100 = 5050$$

Add : $20 + 22 + 24 + \dots + 28 + \dots + 38$

$$= \frac{20+38}{2} \times 10 = 290$$

9] Table 2.

① $\sum_{k=1}^{n} k = 1+2+\dots+n = \frac{(1+n)n}{2}$

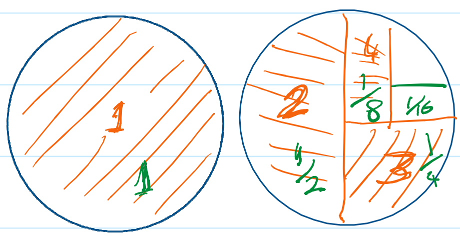
② $\sum_{k=1}^{n} k^2 = 1+4+9+\dots+n^2 = \frac{n(n+1)(2n+1)}{6}$

③ $\sum_{k=1}^{n} k^3 = 1+2^3+3^3+\dots+n^3 = \left(\frac{n(n+1)}{2}\right)^2 = \frac{n^2(n+1)^2}{4}$

④ $\sum_{k=1}^{\infty} x^k, \quad |x|<1 = \frac{1}{1-x}$

10] e.g. Geometric Progression

① $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots < 2$



② $1 + \frac{1}{2} + \dots + \frac{1}{16} = 2 - \frac{1}{16}$

③ $1 + 2 + 4 + 8 + \dots + 32 \quad < \quad 2\cdot(32)$

$= 2\cdot 32 - 1$

④ $\sum_{i=0}^{n} 2^i = 1 + 2 + \dots + 2^n = 2^{n+1} - 1$

11] Geometric Progression

$$\sum_{i=0}^{n} ar^i = \begin{cases} \dfrac{a(r^{n+1}-1)}{r-1} & \text{if } r \neq 1 \\ a(n+1) & \text{if } r = 1 \end{cases}$$

12] e.g.

$100^2 + 101^2 + \dots + 200^2$

$$= \sum_{i=100}^{200} i^2 = \sum_{i=1}^{200} i^2 - \sum_{i=1}^{99} i^2$$

$$= \frac{200(201)(401)}{6} - \frac{99(100)(201)}{6}$$

e.g

$$\sum_{k=0}^{n} \frac{1}{2^k} = 1 + \frac{1}{2} + \frac{1}{4} + \dots + \frac{1}{2^n} = 2 - \frac{1}{2^n}$$