Functions

Wednesday, February 25, 2026 11:09 AM

Recall: ① Quantifiers

② Sets

Missing

700# 19, 28 L .

800#1, 4

1] Exer: which of these are equivalent?

① $\forall x \, (P(x) \wedge Q(x)) \equiv \forall x \, P(x) \wedge \forall x \, Q(x)$ ✓ $\forall$ on $\wedge$

② $\forall x \, (P(x) \vee Q(x)) \equiv \forall x \, P(x) \vee \forall x \, Q(x)$ ✗

③ $\exists x \, (P(x) \wedge Q(x)) \equiv \exists x \, P(x) \wedge \exists x \, Q(x)$ ✗

④ $\exists x \, (P(x) \vee Q(x)) \equiv \exists x \, P(x) \vee \exists x \, Q(x)$ ✓ $\exists$ on $\vee$

5-8 $\forall\exists$ $\boxed{\rightarrow}$ $\emptyset$ $\boxed{\leftrightarrows}$ $\emptyset$

HW Can we distribute $\rightarrow$ / $\leftrightarrow$ on $\wedge$? or on $\vee$? check all.

Soln.

② F   J +B 809

	P	Q
A	T	T
B	T	F
C	F	T

③ F   FM + B 806, 808

	P	Q
A	T	F
B	F	T
C	F	F

④ T   F2 819

2] Exer: if $x \in (A \cup B \cup C) - (B \cap C \cap D)$

then x must be in which set?

	must be	can be
A	No	Yes
B	No	Yes
C	No	Yes
D		No

§ 2.3 Functions

Relation
$R \subseteq A \times B$

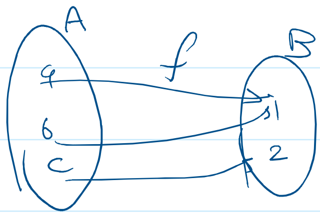
3) Def^n. a relation from A to B is called a function from A to B if : $\forall x \in A$, $\exists! \, y \in B$, $(x,y) \in f$

we write :

$$f : A \longrightarrow B$$

$$f(x) = y \quad \text{iff} \quad (x,y) \in f$$

Here.



A is the domain

B is the codomain

y is the image of x

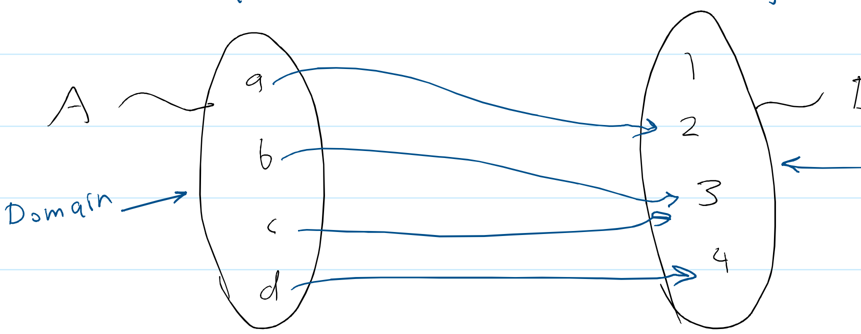
x is the preimage of y

The range of f is $\{ y \in B \mid \exists x \in A , f(x) = y \}$

4) Def^n. the range of S

Let $S \subseteq A$, then $f(S) = \{ y \in B \mid \exists x \in S , f(x) = y \}$

5] e.g. $f = \{ (a,2), (b,3), (c,3), (d,4) \} \subseteq A \times B$



codomain

Range $= \{2,3,4\}$

$f(b) = 3$

- The image of b is 3
- The preimage of 3 is $\{b, c\}$

$f(\{c,d\}) = \{3,4\}$

$f(\{b,c\}) = \{3\}$

6] One-to-one function (injection)

$f: A \to B$ is one-to-one iff $\forall x, y \in A,\ f(x) = f(y) \to x = y$

i.e. $x \neq y \to f(x) \neq f(y)$

7] Onto function (surjection)

$f: A \to B$ is onto iff $\forall y \in B,\ \exists x \in A,\ f(x) = y$

Every $y \in B$ has a preimage

8] Bijection : (one-to-one correspondence)

f is a bijection if it is an <u>injection</u> and <u>surjection</u>

9] e.g. the function f in [5] is neither injection nor surjection $\therefore$ not bijection

10] e.g.

① $f: \mathbb{Z} \to \mathbb{Z},\quad f(x) = x+3$ is a bijection

② $g: \mathbb{Z}^+ \to \mathbb{Z}^+,\quad g(x) = x+3$ is one-to-one
not onto

11] Inverse : of $f: A \to B$ is the function

$f^{-1}: B \to A,\quad f^{-1}(y) = x$ iff $f(x) = y$

12] Theorem : f is invertible if it is a bijection; otherwise, it is not invertible.

e.g. f in eg [10] is invertible

Sol^n.

$f(x) = x+3$

① Let $y = x+3$

② Solve for x:

$\Rightarrow x = y-3$

③ swap variables $x \to f^{-1}(x)$, $y \to x$

$\Rightarrow f^{-1}(x) = x - 3$

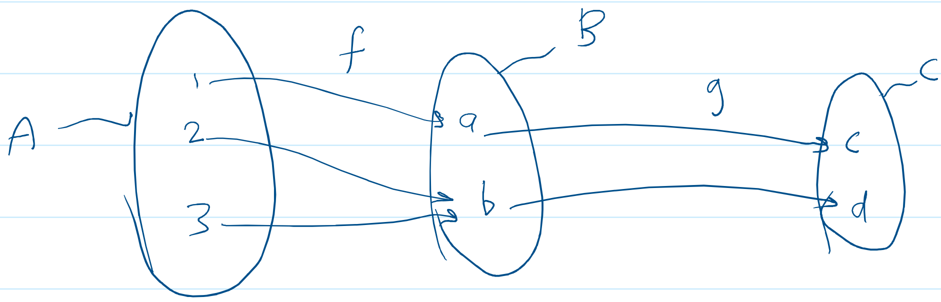
13] Composition

Let $f: A \to B$, $g: B \to C$ be two functions

then the composition $(g \circ f): A \to C$ is a function

defined by: $(g \circ f)(x) = g(f(x))$

e.g.



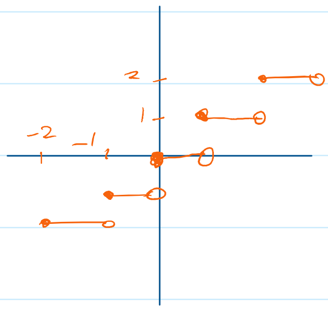
$$g \circ f(2) = g(f(2)) = g(b) = d$$

14] Floor Function :

$f: \mathbb{R} \to \mathbb{Z}$

$f(x) = \lfloor x \rfloor$ is the largest int $\leq x$

e.g. $\lfloor 2.5 \rfloor = 2$



15] Ceiling Function :

$f: \mathbb{R} \to \mathbb{Z}$

$f(x) = \lceil x \rceil$ is the smallest int $\geq x$

e.g.

$\lceil 3.7 \rceil = 4$