Set Operations

Monday, February 23, 2026 11:19 AM

Recall : Sets

Missing:
700 # 5, 18, 19, 23,
800 # 3, 5, 16, 17, 19, 28
+B 724, 725
+B 809, 822

1] Power Set

Given a set A

The power set of A is:

$P(A) = \{ X \mid X \subseteq A \}$

e.g. $A = \{1, 2, 3\}$

$P(A) = \{ \emptyset, \{1\}, \{2\}, \{3\},$
$\{1,2\}, \{1,3\}, \{2,3\},$
$\{1,2,3\} \}$

$X \subseteq A = \{1, 2, 3\}$

$X = \{ \quad \} \neq \{\emptyset\}$

Note:

$\{ \quad \} \neq \{\emptyset\} \neq \emptyset$

$\{\emptyset\} = \{\{ \quad \}\}$

$X \subseteq A = \{1 \; 2 \; 3 \; 4\}$

$X = \{3, 4\} = 0\,0\,1\,1$

2] Thrm:

$$|P(A)| = 2^{|A|}$$

e.g.

$P(\{1\}) = \{\emptyset, \{1\}\}$ // $2^1 = 2$

$P(\emptyset) = \{\emptyset\}$ // $2^0 = 1$

e.g.

$A = \{ \emptyset, \{\emptyset\} \}$

$P(A) = \{ \emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\} \}$

3] Truth set

Def^n. Given a predicate, P, and a domain D,

the truth of P is $\{ x \in D \mid P(x) \}$

e.g. $P(x): \quad x^2 \leq 9$, Domain is $\mathbb{R}^+$

Sol^n: the truth set $P = (0, 3]$

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§ 2.2 Set Operations

4] Operations:

Union $A \cup B = \{x \mid x \in A \vee x \in B\}$

Intersection $A \cap B = \{x \mid x \in A \wedge x \in B\}$

difference $A - B = \{x \mid x \in A \wedge x \notin B\}$

complement $\overline{A} = A' = U - A$

5] Set Identities

① $A \cup \emptyset = A$, $A \cap U = A$ } identity laws

② $A \cup U = U$, $A \cap \emptyset = \emptyset$ } domination laws

③ $A \cup A = A$, $A \cap A = A$ } idempotent laws

④ $A \cap \overline{A} = \emptyset$, $A \cup \overline{A} = U$ } complement laws

⑤ $\overline{(\overline{A})} = A$ Complementation

⑥ $\overline{A \cup B} = \overline{A} \cap \overline{B}$ } De Morgan's laws

⑥ $\overline{A \cup B} = \bar{A} \cap \bar{B}$

$\overline{A \cap B} = \bar{A} \cup \bar{B}$ } De Morgan's laws

⑦ $A \cup (A \cap B) = A$

$A \cap (A \cup B) = A$ } Absorption law

⑧ $A \cap (B \cap C) = (A \cap B) \cap C$

$A \cup (B \cup C) = (A \cup B) \cup C$ } Associative laws

⑨ $A \cup B = B \cup A$ Commutative ...

10 $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ distribution ...

6] Membership table

use 1/0 for T/F

e.g. Prove absorption law: $A \cup (A \cap B) = A$

A	B	$A \cap B$	$A \cup (A \cap B$	$\stackrel{?}{=}$
0	0	0	0	✓
0	1	0	0	✓
1	0	0	1	✓
1	1	1	1	✓

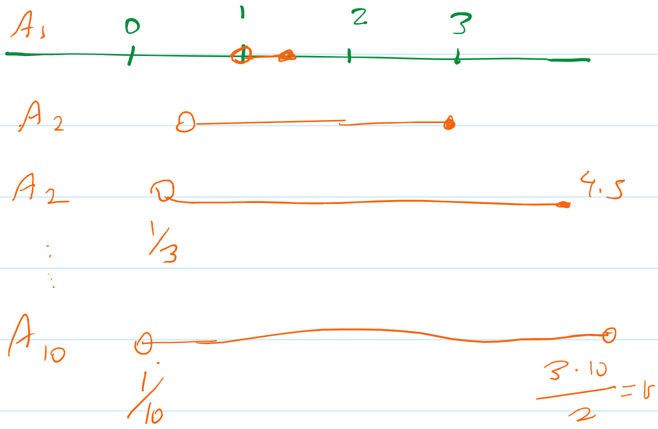
7] Generalized Union and Intersection

① $\bigcup_{i=1}^{n} A_i = A_1 \cup A_2 \cup \cdots \cup A_n$

② $\bigcap_{i=1}^{n} A_i = A_1 \cap A_2 \cap \cdots \cap A_n$

8] e.g. $A_i = (1/i , 3i/2]$

Find $\bigcup_{i=1}^{10} A_i = ?$



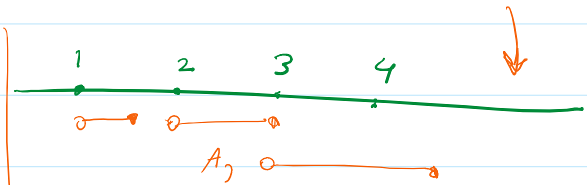
Soln. (F14)

$A_i = (1^{\times} , 15]$ R VVV

$(1/10 , 15]$ ✓ JS +B 809, 822

9] Exer $A_i = (i , 3i/2]$

Find $\bigcup_{i=1}^{\infty} A_i = ?$



Soln.

$(1, 1.5] \cup (2, \infty)$

$(-\infty, \infty)$ ✗ F

$(1, \infty)$ ✗ ✓ JS

$(-\infty, \infty)$ ✗ R

$\emptyset$ ✗ FD

$(0, \infty)$ ✗

10] What are disjoint sets?

Defn.

A and B are disjoint iff $A \cap B = \emptyset$

$A = \{\emptyset, 1, \{1\}\}$

		$\in A$	$\subseteq A$	$\in P(A)$	$\subseteq P(A)$
1	$\{1\}$	✓	✓	✓	✗
2	$\emptyset$	✓	✓	✗	✓