Sets

Wednesday, February 18, 2026 11:23 AM

On Quiz 2

§ 2.1 Sets

1] Def^n. a set is an unordered collection of objects (elements)

2] Set Builder:

$\mathbb{N} = \{0, 1, 2, 3, \dots\}$

$\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$

$A = \{x \mid x \text{ is even}\}$

$\mathbb{Q} = \{a/b : a, b \in \mathbb{Z} \text{ and } b \neq 0\}$

$\mathbb{R}$ = the set of all real numbers

3] Intervals: of real numbers

Notation: $[a, b) = \{x \mid x \in \mathbb{R} \text{ and } a \leq x < b\}$ (closed, open)

4] Membership:

Notation: $x \in A$ means x is an element in A

5] Equality:

$A = B$ iff $\forall x \, (x \in A \leftrightarrow x \in B)$

6] Universal set: U is the set of all elements in the domain.

7] Empty set: $\emptyset = \{\}$

8] Subset : $A \subseteq B$ $\quad \forall x \ (x \in A \longrightarrow x \in B)$

proper subset : $A \subset B$ $\quad \forall x \ ((x \in A \longrightarrow x \in B) \wedge A \neq B)$

$\equiv \forall x (x \in A \rightarrow x \in B) \wedge \exists y (y \in B) \wedge (y \notin A)$

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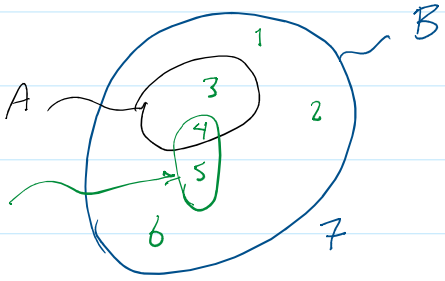
$A = \{3, 4\}$

9] Venn Diagram : $\quad S = \{\quad\}$

- subset $\quad A \subseteq B$
- proper subset $\quad C \subset B$
- common element $\quad (A \cap C)$



$A = \{3, 4\}$

$B = \{1, 2, 3, 4, 5, 6\}$

$C = \{4, 5\}$

$\{1\} \subseteq B$

$\{3\} \subseteq A \ , \ \{4\} \subseteq A$

$\{3, 4\} \subseteq A \ , \ \emptyset \subseteq A$

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10] Thrm :

$\emptyset \subseteq S$ and $S \subseteq S$ for all sets

11] Exer (F13)

$A = \{3, 4\}$

$B = \{1, 2, 3, 4, 5, 6\}$

$C = \{4, 5\}$

$D = \{\{1\}\}$

$E = \emptyset$

	$\subseteq A$	$\subseteq B$	$\subseteq C$	$\subseteq D$
	yes	yes	No	No
	No	yes	No	No
	No	yes	yes	No
	No	No	No	yes
	yes	yes	yes	yes

$D = \{\{1\}\}$	No	No	No	Yes
$E = \emptyset$	yes	Yes	yes	Yes
$F = \{4\}$	yes	Yes	yes	No
$G = \{1\}$	No	Yes	No	No

Exer (F14)

Let $F = \{1, \{2,3\}, \{4, \emptyset\}\}$. $G = \{\{1, \{2\}\}, \{\}\}$

		$\in F$	$\subseteq F$	$\in G$	$\subseteq G$
$A = \{1\}$	FR →	No ~~yes~~ ✗	Yes ~~No~~ ✗	No ✓	~~Yes~~ No
$B = \{\{1\}\}$	FA →	No ✓	No ~~Yes~~ ✗	No ~~Yes~~	~~Yes~~ No
+B826 $C = \{1, \{2\}\}$	W →	No ✓	No ✓	Yes ✓	No ✓
+B809 806 $D = \{\{2\}\}$	JM → ✓	No ✓	No ✓	No ✓	No ✓
$E = \{\emptyset\}$	kF →	No ✓	~~Yes~~ No	No ~~Yes~~ ✗	Yes ✓
$\emptyset$	Z → ✓	No ✓	~~yes~~ ✓	~~No~~ Yes	Yes ✓

12] Cartesian Product (ordered pair)

$$A \times B = \{ (a,b) \mid a \in A \wedge b \in B \}$$

e.g. $A = \{1, 2, 3\}$, $B = \{4, 5\}$, $C = \{5, 6\}$

$$A \times B = \{ (1,4), (2,4), (3,4), (1,5), (2,5), (3,5) \}$$

$$B \times C = \{ (4,5), (5,5), (4,6), (5,6) \}$$

13] Set cardinality : The size of set

Notation : $|A|$ = number of elements in A

e.g $A = \{4, 5, 6\}$

$|A| = 3$

14] Thrm :

$$|A \times B| = |A| \cdot |B|$$

15] Sets and Quantifiers

$$\forall x \in \mathbb{Z}^+ \quad (x^2 \geq x)$$ (math notation)