Proofs

Monday, February 16, 2026 11:16 AM

Recall : Rules of Inference

on CA1
700# none
800# 1, 11, 13, 23,
+B # 733, 822
Ex #711 on 2/11

1) Exer
1. a student in this class has not read the book.
2. Everyone in this class passed the exam.
∴ 3. Someone passed the exam and did not read the book.

Proof:

1. $\forall x \; C(x) \to S(x)$ premises
2. $\forall x \; (C(x) \wedge S(x) \to P(x))$
3. $C(\text{Fatima})$
4. $C(\text{Fatima}) \to S(\text{Fatima})$ Universal Instantiation (1)
5. $C(\text{Fatima}) \wedge S(\text{Fatima}) \to P(\text{Fatima})$ Univ. Inst. (2)
6. $S(\text{Fatima})$ Modus Ponens (3, 4)
7. $C(\text{Fatima}) \wedge S(\text{Fatima})$ Conjunction (3, 6)
8. $P(\text{Fatima})$ Modus Ponens (5, 7)

∴ Valid

2] Universal Modus Ponens

$$\begin{array}{l} \forall x \; P(x) \to Q(x) \\ P(a) \\ \hline \therefore Q(a) \end{array}$$

3] Universal Modus Tollens

$$\forall x \; P(x) \longrightarrow Q(x)$$
$$\neg Q(a)$$
$$\therefore \neg P(a)$$

5] Def^n. even number : n is even if $\exists k \in \mathbb{Z}, n = 2k$

odd numbers : n is odd if $\exists k \in \mathbb{Z}, n = 2k+1$

rational number : $x \in \mathbb{Q}$ if $\exists a, b \in \mathbb{Z}, b \neq 0, x = a/b$

§ 1.7 Proofs

4] Def^n. Even numbers : n is even iff $\exists k \in \mathbb{Z}, \; n = 2k$.

Odd numbers : n is odd iff $\exists k \in \mathbb{Z}, \; n = 2k+1$.

Rational numbers : $x \in \mathbb{Q}$ iff $\exists a, b \in \mathbb{Z}, \; b \neq 0, \; x = a/b$

5] Proof Methods :

① Direct Proof : to prove $p \to q$

we show $p \to r$

$\to \ldots$

$\to q$

② Indirect Proof (conraposition)

to prove $p \to q$

we prove $\neg q \longrightarrow \neg p$

assume q is false , then p is false

③ Prove by Contradiction

to prove that p is true

6] Direct proof :

e.g. prove that n is odd then n^2 is odd

proof :

let $n = 2k+1$, for $k \in \mathbb{Z}$, by def^n.

$\Rightarrow n^2 = (2k+1)^2$

$= 4k^2 + 4k + 1$

$= 2(2k^2+2k)+1$; $(2k^2+2k) \in \mathbb{Z}$

$\Rightarrow n^2$ is odd by def^n.

7] Indirect Proof

Prove that : if $3n+2$ is odd, then n is odd

Try direct proof:

let $3n+2 = 2k+1$ by def^n

$3n = 2k-1$

can go nowhere...

Proof:

Assume n is even

let $n = 2k$, $k \in \mathbb{Z}$, by def^n

$\Rightarrow 3n+2 = 3(2k)+2$

$= 2(3k+1)$; $(3k+1) \in \mathbb{Z}$,

$\Rightarrow 3n+2$ is even by def^n

$\therefore$ if $3n+2$ is odd, then n is odd by contraposition.

8] Proof by Contradiction

to prove : P is true

Assume $\neg p$ (p is false)

then show a contradiction

e.g. Prove that $\sqrt{2}$ is irrational

Proof: Assume $\sqrt{2}$ is rational

let $\sqrt{2} = a/b$, for $a, b \in \mathbb{Z}$, $b \neq 0$

in the simplest form (no common factors)

$\Rightarrow 2 = a^2/b^2$

$\Rightarrow 2b^2 = a^2$ $\Rightarrow a^2$ is even by def^n

$\Rightarrow a$ is even

$\Rightarrow \exists c,\ a = 2c$ by def^n

$\Rightarrow 2b^2 = (2c)^2$

$= 4c^2$

$= 2(2c^2)$ $\Rightarrow b^2$ is even by def^n

$\Rightarrow b$ is even

$\therefore$ a and b have a common factor

This contradicts the assumption.

$\therefore \sqrt{2}$ is irrational.

9] Vacous and Trivial Proofs

to prove : $p \rightarrow q$

<u>Vacous proof:</u> show that p is false

<u>Trivial Proof</u>: show that q is true

10] Proof of equivalence :

① to prove $p \leftrightarrow q$

prove $p \to q$

and $q \to p$

② to prove that p, q, and r a equivalent

just prove that $p \to q$,

$q \to r$, and

$r \to p$