Casting Subdomains

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Recall : Rules of Inference | Missing

1] Exer : Prove that this argument is valid

① Lynn works part time or full time. (p, f)
② If Lynn does not play on the team (t), then she does not work part time.
③ If Lynn plays on the team, she is busy. (b)
④ Lynn does not work full time.
∴ Therefore, Lynn is busy. $\therefore b$

Proof

1. $p \vee f$
2. $\neg t \longrightarrow \neg p$
3. $t \longrightarrow b$
4. $\neg f$

} premises

5. p — Disj. Syllogism (1, 4)
6. $\neg(\neg t)$ — M.T. (2, 5)
7. t — Double negation (6)
8. b — M.P. (3, 7)

∴ It is valid. $\square$

2] Casting Subdomains

eg. The domain : all KFUPM students

$C(x)$: x is this class

$S(x)$: x is smart

① Everybody in this class is smart

$\forall x \; S(x) \wedge C(x)$ ✗

$\forall x \; C(x) \rightarrow S(x)$ ✓

② Someone in this class is smart

$C(x) \rightarrow \exists x \; S(x)$ ✗ J

$\exists x \; C(x) \wedge S(x)$ W +B

$\exists x \; \underline{S(x)} \rightarrow C(x)$ ✗ F

$\exists x \; \underbrace{\underbrace{C(x)}_{} \rightarrow \underbrace{S(x)}_{}}_{F}$ ✗ F

T, for $x = a$

3] Note: To cast subdomain C of the main domain D:

① For all x in C: $\underbrace{\forall x \; C(x) \rightarrow}_{\text{everyone in this class}} \; P(x \ldots)$

② For some x in C: $\underbrace{\exists x \; C(x) \wedge}_{\text{Someone in this class}} \; P(x \ldots)$

③ Avoid: $\exists$ and $\rightarrow$

it is meaningless and confusing.

4] e.g.

The domain is $\mathbb{R}$

① $\exists x \; (x^2 < x)$

T, for $x = 0.5$

② $\exists x \quad (x > 1) \rightarrow (x^2 < x)$

T, for $x = -3$

③ $\forall x \quad (x > 1) \rightarrow (x^2 < x)$

F, $x = 2$ $\quad x = -2$

5] Rules of Inference for Quantifiers

let D be the domain

① $\dfrac{\forall x \; P(x)}{\therefore P(m)}$ for any $m \in D$ Universal Instantiation

② $\dfrac{P(a)}{\therefore \forall x \; P(x)}$ for an arbitrary $a \in D$ Universal Generalization

③ $\dfrac{\exists x \; P(x)}{\therefore P(c)}$ for some $c \in D$ Existential Instantiation

④ Existential Generalization

$\dfrac{P(c)}{\therefore \exists x \; P(x)}$ for some $c \in D$

6] e.g.

1. a student in this class has not read the book.
2. Everyone in this class passed the exam.

$\therefore$ 3. Someone passed the exam and did not read the book.

Sol^n.

$C(x)$: x is in this class.

$B(x)$: x read the book.

$P(x)$: x passed the exam.

Proof:

1. $\exists x\,(C(x) \rightarrow \neg B(x))$ premise
2. $\forall x\,(C(x) \rightarrow P(x))$ Premise
3. $C(a) \wedge \neg B(a)$ Exist. Inst. (1) for some $a \in D$
4. $C(a)$ Simplification (3)
5. $C(a) \longrightarrow P(a)$ Univ. Inst. (2)
6. $P(a)$ Modus Ponens (4,5)
7. $\neg B(a)$ Simplification (3)
8. $P(a) \wedge \neg B(a)$ Conjunction (6,7)
9. $\exists x\, P(x) \wedge \neg B(x)$ Existential Generalization (8)

$\square$