Rules of Inference

Monday, February 9, 2026 11:12 AM

Missing:
700 # 7, 19,
800 # 3, 23, 25,

1) Exer

- Domains:
  - X = all ics253 students
  - Q = all questions in an ics253 exam
- p(x, q) = student x solved question q
- Write the following in symbolic notation:

"There is exactly one student who got full mark."

Solution:

F14:

① $\exists x \; \forall q \; P(x, q)$ FR

② $\exists x \; \forall q \; P(x, q) \wedge x = x_1$ MF

F13:

① $\exists x \; \forall q \; P(x, q)$

Some people got full mark

② $\forall q \; \exists x \; P(x, q)$ ② $\xrightarrow{\text{negate}}$ $\exists q \; \forall x \; \neg P(x, q)$

There is no hard question

③ $\exists x \; \forall q \; P(x, q) \wedge \underline{\exists q \; \forall x} \; \neg P(x, q)$

Someone got full mark and there is no hard question

Sol^n.

$$\exists x \left[\left(\forall q \; P(x,q)\right) \wedge \underbrace{\forall z \left(\forall q \left(P(z,q) \right) \longrightarrow z = x \right)}_{\text{contrapositive}} \right]$$

q

2] Uniqueness quantifier

Notation: $\exists! x$ exactly one x in the domain

e.g.- Exactly one student got full mark

$\exists! x \; \forall q \; P(x,q)$

$z \neq x \longrightarrow$

§ 1.6 Rules of Inference

3] Key words:

argument, $\longrightarrow$ e.g.

valid

$$\left[\begin{array}{ll} p \longrightarrow q & \text{Premises} \\ p & \text{(hypothesis)} \\ \hline \therefore q & \text{conclusion} \end{array} \right.$$

4] Rules of inference

①

$$\begin{array}{l} p \\ p \longrightarrow q \\ \hline \therefore q \end{array}$$

Modus Ponens

Sentential form (Proposition)

$[p \wedge (p \to q)] \longrightarrow q$ is a tautology

(2)

$\neg q$
$p \to q$

$\therefore \neg p$

Modus Tollens

$[\neg q \wedge (p \to q)] \to \neg p$ is a tautology.

(3)

$p \to q$
$q \to r$

$\therefore p \to r$

Hypothetical Syllogism

(4)

$p \vee q$
$\neg p$

$\therefore q$

Disjunctive Syllogism

(5)

p

$\therefore p \vee q$

Addition

(6)

$p \wedge q$

$\therefore p$

Simplification

(7)

p
q

$\therefore p \wedge q$

Conjunction

(8)

$p \vee q$
$\neg p \vee r$

$\therefore q \vee r$

Resolution

$[(p \vee q) \wedge (\neg p \vee r)] \to q \vee r$ is a tautology.

5] e.g.

If you send me an email,
I will finish the program.
If you don't send me an email,
I will go to bed early. — r
If I go to bed early,
I will wake up feeling refreshed — s

∴ If I don't finish the program,
I will wake up feeling refreshed. $\neg q \to s$

Prove that this argument is valid.

Solⁿ.

hypothesis
$$\begin{cases} p \to q \\ \neg p \to r \\ r \to s \end{cases}$$

Conclusion $\therefore \neg q \to s$

Proof:

1.	$p \to q$	hypothesis
2.	$\neg q \to \neg p$	Contrapositive ①
3.	$\neg p \to r$	hypothesis
4.	$\neg q \to r$	hypothetical Syllogism (2,3)
5.	$r \to s$	hypo.
6.	$\neg q \to s$	hypothetical Syllogis (4,5)

∴ valid argument.

6] Exer:

$$[(p \vee q) \wedge (\neg p \vee q)] \to q \vee r$$

— p

— q

p	q	r	$p \vee q$	$\neg p \vee r$	$\wedge$	$q \vee r$	$\to$
T	T	T	T	T	T	T	T

T	T	T	T	T	T	T	T
T	T	F	T	F	F	T	T
T	F	T	T	T	F	T	T
T	F	F					T
F	T	T					T
F	T	F					T
F	F	T					T
F	F	F					T

7) Exer: $[(p \vee q) \wedge (\neg p \vee q)] \longrightarrow q \wedge r$

p	q	r	$p \vee q$	$\neg p \vee r$	$\wedge$	$q \wedge r$	$\rightarrow$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	T	T	T	F	F
T	F	F					
F	T	T					
F	T	F					
F	F	T					
F	F	F					

∴ not tautology.

8) Fallacies:

a fallacy is a mistake or illogical step in the proof.
(Learn them to avoid)

① $p \longrightarrow q$
q
―――――
$\therefore p$

Affirming the conclusion

② $p \to q$

$\neg p$

$\therefore \neg q$

Denying the hypothesis

③ Circular reasoning (begging the question).

The statement is proved using itself.