Nested Quantifiers Exercises

Wednesday, February 4, 2026 11:17 AM

Recall: Quantifiers

on Quiz 1
700#
+1B# 24, 6, 28, 31, 20
800#
+B # 6, 8, 7, 9, 26, 17, 25
+2B # 732, 804, 823

1] Negation

e.g. $P(x)$: x is a black horse
$Q(x)$: x is a white horse

$\forall x \quad P(x)$ domain: $H = \{1, 2, 3\}$

$\equiv P(1) \wedge P(2) \wedge P(3)$

$\neg \forall x \quad P(x) \equiv \neg (P(1) \wedge P(2) \wedge P(3))$

$\equiv \neg P(1) \vee \neg P(2) \vee \neg P(3)$ by DeMorgan's

$\equiv \exists x \neg P(x)$

$\therefore$ $\neg \forall x \; P(x) \equiv \exists x \; \neg P(x)$

$\neg \exists x \; P(x) \equiv \forall x \; \neg P(x)$

2] Nested Quantifiers as loops

$\forall x \; \forall y \quad P(x, y)$

for $x = 1$ to n
  for $y = 1$ to m

    $P(x, y) = T$

P	y=1	2	3 ... m
x = 1	T	T	T
2	T	T	...
3	⋮	⋮	
⋮	⋮	⋮	
n			

3] Quantifiers of two variables

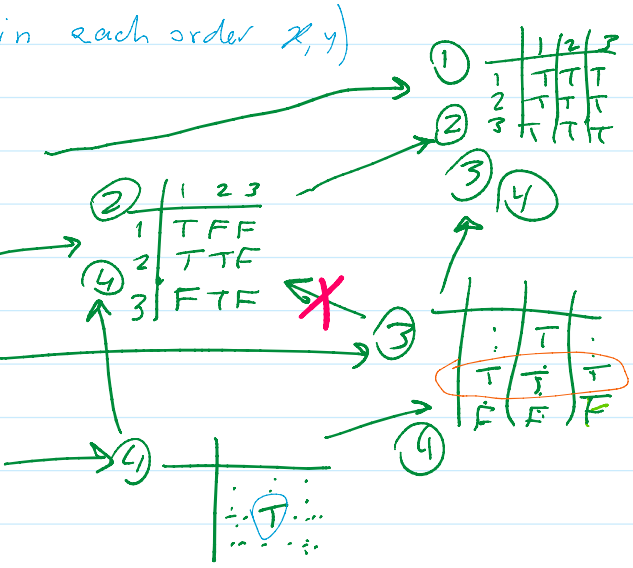
4 - cases (in each order x, y)

① $\forall x \; \forall y \quad P(x,y)$

② $\forall x \; \exists y \quad P(x,y)$

③ $\exists x \; \forall y \quad P(x,y)$

④ $\exists x \; \exists y \quad P(x,y)$



4] Exer

T/F and justify. Domain = $\mathbb{R}$

① $\forall x \; \exists y \quad x + y = 0$

T, for $y = -x$

② $\forall x \; \exists y \quad x \cdot y = 1$

T, for $y = \frac{1}{x}$

F, for $x = 0$

③ $\forall x \; \forall y \; \exists z \quad (x \leq z) \wedge (y \geq z) \equiv (x \leq z \leq y)$

① T, for $z = y - 1$, $y = x + 2$

② T, for $z = \frac{x+y}{2}$

③ T, for $z = y = x$ ✓✓✓

④ F, for $x > y$ ✗

⑤ T, for $z = y$, —— ✗

⑥ F, for $x = y + 2$

⑦ F, for $x = z + 1$

⑧ F, $x = 5$ → $y = 2$ ✓

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