Predicates and Quantifiers

Wednesday, January 28, 2026 10:57 AM

Recall : Logical Equivalence
Applications : K&K puzzles

Missing
Zoom # 1, 3, 6,
Room 3, 25,

1] Exer. K&K

You meet 3 people

A : we are all knights
B : if A is a knight, then C is a knave
C : B is a knave

$A \leftrightarrow A \wedge B \wedge C$
$B = A \rightarrow \neg C$
$C \rightarrow \neg B$

① Truth table method $\Longrightarrow$ 8 rows

② Tree Method

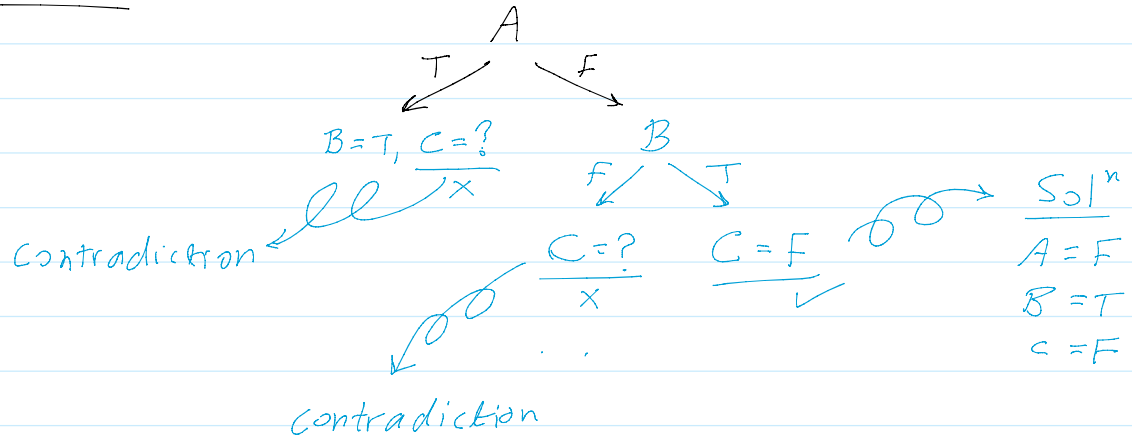
Soln. by JW

A	B	C	
F	T	F ✓	JW/SS

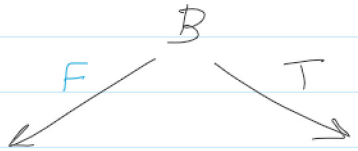
Soln. by Z, M

A	B	C	
F	F	T	X
T	F	T	X

Tree Method



another soln.
start at B



A=F, C=? → ✗ → contradiction

A=F, C=F ✓ → Soln.

By Truth table

① $A \overset{=}{\longleftrightarrow} A \wedge B \wedge C$

② $B = A \rightarrow \neg C$

③ $C = \neg B$

A	B	C	$A \wedge B \wedge C \leftrightarrow A$ ①	②	③
T	T	T	T		F
T	T	F	T	T	F
⋮					
F	T	F	T	T	T

§ 1.4 Predicates and Quantifiers

2] e.g. Every student in my class is SWE.

This true only if all students are SWE.

3] Notation:

$\forall x \quad P(x)$

upper case

predicate (Propositional function)
e.g $P(x)$ = x is SWE

variable in the domain D
$x \in D$; e.g. D is our class

quantifire

4] Universal Quantifier

$\forall x$ means for all elements in the domain

5] Existential Quantifier:

$\exists x$ means for some element in the domain

6] e.g. Let the domain be all students in my class
$P(x)$ means x is SWE

① $\forall x \; P(x)$

Soln. (F14) True, for x = Maha (SWE) ✓ ✗
→ False ✓, for x = Wu (AICS) ✓
True ✗, for x = Fatima (COE) ✗
False ✓, for x = Maha ✗

$\forall x, \; x^2 > x$ False, because some numbers get small when we square them

False, for $x = \frac{1}{2}$

Soln. (F13) False ✓, for x = Zainab (CS) ✓
True ✗, for x = Layan (SWE) ←
False ✓, for x = Habibah (SWE) ✓ ✗
True ✗, for x = Deemah (CS) ✗

② $\exists x \quad P(x)$

Soln. (F14) · T, for x = Maha ✓
F, for x = Fatima (COE)

Soln. (F13) True ✓, for x = Layan ✓
False ✗, for x = Deemah ✓✗
True ✓, for any SWE student ✗ why?
See the Red Table

7] The Red Table

	True	False
$\exists x$	Example	Proof
$\forall x$	Proof	counterexample

8] Exer.

$\forall x \quad \forall y \quad (x > y)$ Domain = $\mathbb{R}$

F, $x = 2$ $y = 3$ ✓

T, $x = 5$ $y = 2$

$\forall x \quad \exists y \quad (x > y)$

T, for $y = x - 1$

F, for $x = -100$