Propositional Equivalence

Saturday, January 24, 2026 5:58 PM

§1.3 Propositional Equivalence

1] Logical Equivalence (Table 6)

① $p \wedge T \equiv p$
$p \vee F \equiv p$ } indentity laws

② $p \vee T \equiv T$
$p \wedge F \equiv F$ } domination laws

③ $p \vee p \equiv p$
$p \wedge p \equiv p$ } idempotent laws

④ $\neg(\neg p) \equiv p$ double negation

⑤ $p \vee q \equiv q \vee p$
$p \wedge q \equiv q \wedge p$ } commutative laws

⑥ $(p \vee q) \vee r \equiv p \vee (q \vee r)$
$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$ } associative Laws

⑦ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ } distribution Laws

⑧ $\neg(p \wedge q) \equiv \neg p \vee \neg q$
$\neg(p \vee q) \equiv \neg p \wedge \neg q$ } De Morgan's Laws

⑨ $p \wedge (p \vee q) \equiv p$
$p \vee (p \wedge q) \equiv p$ } Absorption Laws

⑩ $p \vee \neg p \equiv T$
$p \wedge \neg p \equiv F$ } Negation laws

2] Conditional Laws (Table 7)

① $p \to q \equiv \neg p \vee q$ conditional law

② $p \to q \equiv \neg q \to \neg p$ Contrapositive

③ $p \vee q \equiv \neg p \to q$ conditional law

④ $p \wedge q \equiv \neg(p \to \neg q)$

⑤ $(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$

⑥ $(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$

⑦ $(p \to q) \vee (p \to r) \equiv p \to (q \vee r)$

⑧ $(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$

3] Biconditional Laws (Table 8)

① $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

$\equiv (\neg p \vee q) \wedge (\neg q \vee p)$

$\equiv (p \wedge q) \vee (\neg p \wedge \neg q)$

② $\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

4] e.g. Show that $\neg(p \to q) \equiv p \wedge \neg q$ without truth ~~table~~.

proof.

$LHS = \neg(p \to q) \equiv \neg(\neg p \vee q)$ by conditional law

$\equiv \neg(\neg p) \wedge \neg q$ by De Morgan's

$\equiv p \wedge \neg q$ by double negation

$\equiv RHS$ $\square$

5] e.g. Show that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology.

proof:

$$(p \wedge q) \rightarrow (p \vee q)$$

$\equiv \neg(p \wedge q) \vee (p \vee q)$	conditional law
$\equiv (\neg p \vee \neg q) \vee (p \vee q)$	DeMorgan's
$\equiv (\neg p \vee p) \vee (\neg q \vee q)$	commutative and assoc. Laws
$\equiv T \vee T$	negation Law
$\equiv T$	domination law.

6] Satisfiability Problem: (SAT) $\in$ NP-Complete

Given a logical expression

$$f = (x_1 \wedge x_2 \wedge \neg x_3) \vee (x_2 \wedge \neg x_4 \wedge \neg x_1) \vee \dots \vee (x_7 \wedge x_n)$$

is f satisfiable?

7] How to solve knight and knave problems?

Truth table	Tree Method
• long	• fast
• gives all solutions	• gives one solution

Notation

A means A is True (Knight)

$\neg A$ means A is a knave (False)

B says "X" we write $B \underset{B = X}{\overset{\text{says}}{\longleftrightarrow}} X$

e.g.

· A says " Either I'm a knave or B is a knight

$$A \longleftrightarrow \neg A \vee B \quad \text{——} \ (1)$$

Truth Table Method

A	B	$\neg A \vee B$	$A \overset{(1)}{\longleftrightarrow} \neg A \vee B$
T	T	T	T
T	F	F	F
F	T	T	F
F	F	T	F

$\rightsquigarrow$ A is a knight
B is a knave

Tree Method :

A

F: $\neg A \vee B$ ⇒ Contradiction

T: $\neg A \vee B \rightsquigarrow$ A is knight
B is knave