Applications

Wednesday, January 21, 2026 11:22 AM

Recall: Propositional Logic

§ 1.2 Applications (cont...)

1] Exer.

If you visit me, you drink coffee, unless you are sick.

V : visit me
C : drink coffee
S : you are sick

p unless q
$\neg q \longrightarrow p$

Soln.

① $V \wedge \neg S \longrightarrow C$ ✓ JW / FRB
② $V \longrightarrow (\neg S \longrightarrow C)$ AA
③ $(V \longrightarrow C) \vee S$ EM
④ $V \vee C \longrightarrow \neg S$ DN

Soln. $\boxed{\neg S \longrightarrow (V \longrightarrow C)}$

① $V \wedge \neg S \longrightarrow C$ ✓ HNR
② $S \longrightarrow (V \longrightarrow C)$ ZZ
③ $(V \wedge C) \longrightarrow \neg S$ GR
④ $(S \wedge V) \longrightarrow \neg C$ RD

$\Longrightarrow$ $\neg S \longrightarrow (V \longrightarrow C)$

$\equiv S \vee \neg V \vee C$ by conditional law $p \to q \equiv \neg p \vee q$

$\equiv \neg(S \vee \neg V) \longrightarrow C$ by conditional Law

$\equiv (\neg S \wedge V) \longrightarrow C \equiv$ ① by De Morgan's law

$\neg(C \vee \neg V) \rightarrow S$

$\equiv (\neg C \wedge V) \rightarrow S$?

④ $(S \wedge V) \rightarrow \neg C$

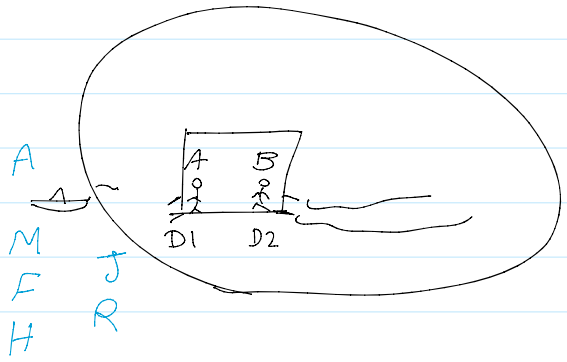
$\equiv \neg(S \wedge V) \vee \neg C$ by conditional

$\equiv \neg S \vee \neg V \vee \neg C$ by De Morgan's

2] knights & knaves Puzzles

knight : T

knave : F



H: A, are you a man and D1 is safe?
A: yes, H takes D1 ×

Soln. A, if I ask B is D1 safe, what would he say?

3] e.g. A: We are both knights
B: A is a knave.

Soln. A is knave, B is knight

4] Exer. A: at least one of us is a knave.
B: says nothing (sleeping)

Soln. A is a knight, B is a knave

§ 1.3 Propositional Equivalence

5] Def^{n}.

	e.g.
① tautology	$p \vee \neg p$
② contradiction	$p \wedge \neg p$
③ Contingency	$p \wedge q$
④ Satisfiable	$p \wedge q$, $p \vee \neg p$

6] Logical Equivalence :

p and q are equivalent if $p \leftrightarrow q$ is a tautology.

e.g.

$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (q \vee r)$$

How to prove it ?

① use truth table

② use logical equivalence laws

To prove $p \equiv q$, we show that $p \leftrightarrow q$ is a tautology