Propositional Logic

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§ 1.1 Propositional logic

1] Proposition:

variable : e.g. p : It is raining

w : the floor is wet

2] Logical operation:

Conjuction : $p \wedge q$

disjunction : $p \vee q$

negation : $\neg p$

3] Exclusive - or :

$$p \oplus q = (p \vee q) \wedge \neg (p \wedge q)$$

Truth table

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

4] Conditional Statement

$p \rightarrow q$:

Truth table

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

if p, then q

p implies q

p is sufficient for q

q is necessary for q

p only if q

q if p

q when p

5] Biconditional Statement

$p \longleftrightarrow q$: p if and only if q

p is necessary and sufficient for q

6] Precedence rule

$\neg, \wedge, \vee, \rightarrow, \leftrightarrow$

e.g. Evaluate :

① $p \wedge q \vee r \rightarrow p \vee q \wedge r$ for $p = T, q = r = F$.

Solⁿ: $\equiv [(p \wedge q) \vee r] \rightarrow [p \vee (q \wedge r)]$

$\equiv [(T \wedge F) \vee F] \rightarrow [T \vee (F \wedge F)]$

$\equiv [F \vee F] \rightarrow [T \vee F]$

$\equiv F \rightarrow T$

$\equiv T$

② $p \vee q \wedge r \rightarrow q \wedge p \vee r$ for $p = T, q = F, r = F$

$\equiv T \vee (F \wedge F) \rightarrow (F \wedge T) \vee F$

$\equiv T \vee F \rightarrow F \vee F$

$\equiv T \rightarrow F$

$\equiv F$

7] Logical Equivalence :

$p \rightarrow q \equiv \neg p \vee q$ conditional law

$\equiv q \vee \neg p$

$\equiv \neg q \rightarrow \neg p$

$\therefore \boxed{p \rightarrow q \equiv \neg q \rightarrow \neg p}$ Contrapositive

$p \rightarrow q \not\equiv \neg p \rightarrow \neg q$ inverse

$p \rightarrow q \not\equiv q \rightarrow p$ converse

$\} \not\equiv p \rightarrow q$

8] Unless : means "if not"

e.g. ① I walk to work unless I have a car

w : I walk to work
c : I have a car

Soln. I walk to work "if not" have a car

$\neg c \rightarrow w$

§ 1.2 Applications

9] Translate English to Logic notation

e.g. ① w : walk to work
c : I have car
"I will walk to work unless I have a car"

Soln.
$\neg c \rightarrow w$

② p : play golf
r : it is raining
b : I'm busy

"I play golf if I am free unless it is raining"

Soln.

① $\neg(r \wedge b) \rightarrow p$ JW ✓

② $\neg(b \vee r) \rightarrow p$ AN ✓

③ $\neg r \rightarrow (p \wedge \neg b)$ JD ✗

④ $\neg r \wedge \neg b \rightarrow p$ NF ✓

Soln. $\neg r \rightarrow (\neg b \rightarrow p)$

$\equiv \neg r \to (b \vee p)$ } by conditional law

$\equiv \underline{r \vee b} \vee p$

$\equiv \neg(r \vee b) \to p$ by conditional law

$\equiv \neg r \wedge \neg b \to p$ by De Morgan's law

10] Exer: Are these statements equivalent?

#1 ① If you study, you pass.

② You pass unless you don't study.

③ You pass only if you study.

Sol^n.

① $s \to p$

② $\neg(\neg s) \to p \equiv s \to p \equiv$ ①

③ $p \to s$ not equivalent

11] Consistent Statements:

No conflict, no contradictions

e.g. System Specs:

① Emails are saved or forwarded.

② If an email is not saved then it is forwarded.

③ Emails are not saved.

Are these consistent?

Sol^n. yes. why?

12] Logical Puzzles

① Muddy Children